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ENGINEERING AND DESIGN

DESIGN OF STRUCTURES TO RESIST THE EFFECTS OF ATOMIC WEAPONS

BURIED AND SEMIBURIED STRUCTURES
# DESIGN OF STRUCTURES TO RESIST THE EFFECTS OF ATOMIC WEAPONS
## BURIED AND SEMIBURIED STRUCTURES

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BIBLIOGRAPHY
ENGINEERING AND DESIGN

DESIGN OF STRUCTURES TO RESIST THE EFFECTS OF ATOMIC WEAPONS
BURIED AND SEMIBURIED STRUCTURES

INTRODUCTION

11-01 PURPOSE AND SCOPE. This manual is one in a series issued for the
guidance of engineers engaged in the design of permanent-type military
structures required to resist the effects of atomic weapons. It is appli-
cable to all Corps of Engineers activities and installations responsible
for the design of military construction.

The material is based on the results of full-scale atomic tests and
analytical studies. The problem of designing structures to resist the ef-
facts of atomic weapons is new and the methods of solution are still in the
development stage. Continuing studies are in progress and supplemental
material will be published as it is developed.

The methods and procedures were developed through the collaboration
of many consultants and specialists. Much of the basic analytical work was
done by the engineering firm of Ammann and Whitney, New York City, under
contract with the Chief of Engineers. The Massachusetts Institute of Tech-
nology was responsible, under another contract with the Chief of Engineers,
for the compilation of material and for the further study and development
of design methods and procedures.

It is requested that any errors and deficiencies noted and any sug-
gestions for improvement be transmitted to the Office of the Chief of Engi-
eers, Department of the Army, Attention: ENGB.

11-02 REFERENCES. Manuals - Corps of Engineers - Engineering and Design,
containing interrelated subject matter, are listed as follows:

DESIGN OF STRUCTURES TO RESIST THE EFFECTS
OF ATOMIC WEAPONS

EM 1110-345-413 Weapons Effects Data
EM 1110-345-414 Strength of Materials and Structural Elements
EM 1110-345-415 Principles of Dynamic Analysis and Design
a. References to Material in Other Manuals of This Series. In the text of this manual references are made to paragraphs, figures, equations, and tables in the other manuals of this series in accordance with the number designations as they appear in these manuals. The first part of the designation which precedes either a dash, or a decimal point, identifies a particular manual in the series as shown in the table following.

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b. Bibliography. A bibliography is given at the end of the text. Items in the bibliography are referenced in the text by numbers inclosed in brackets.

c. List of Symbols. Definitions of the symbols used throughout this manual series are given in lists following the table of contents in EM 1110-345-413 through EM 1110-345-416.

11-03 RESCISSIONS. Draft EM 1110-345-421 (Part XXIII - The Design of Structures to Resist the Effects of Atomic Weapons, Chapter 11 - Buried and Semiburied Structures).

GENERAL

11-04 TYPES OF BURIED STRUCTURES. Underground structures may be divided into two principal categories. These are buried and semiburied structures as illustrated in figure 11.1. Buried structures are those which are
constructed completely below grade elevation. The surface of the earth cover above them is at or near the original grade elevation flush with the surrounding ground surface.

Semiburied structures are those which are placed with a portion or all but the foundation of the structure located above grade elevation. The earth cover over the structure is placed above the original grade elevation.

A comparison of the relative advantages of buried versus semiburied structures indicates that the dynamic loading is more severe in some cases for semiburied than for buried structures. However, semiburied structures require less excavation, are easier to provide with accessways, and are simpler to drain.

A further division of underground structures into rectangular and shell types of construction is provided for the purpose of pointing out important differences in the design procedure. Under uniform loading the curved elements in the shell type of construction are very rigid and are designed by static design methods, whereas the linear elements of rectangular structures are relatively flexible and are proportioned using dynamic design methods.

**11-05 DESIGN FOR EFFECTS OF AIR BURST.** Buried structures are subjected to dynamic earth pressures when exposed to air- or surface-burst atomic weapons. These pressures, which vary with time, position, and shape of structure and size and position of the weapon, are discussed in EM 1110-345-413 and are assumed to be unaffected by the motion of the structure, either as a unit or in some or all of its elements. However, the effect of the earth mass on the dynamic behavior of the walls and roof elements is accounted for in an empirical manner. Based on tests made in Nevada with buried
structures subject to an air-burst atomic weapon [3], it appears reasonable to assume that a rectangular mass of earth of thickness equal to the depth of the earth cover as shown in figure 11.2 moves with the roof of a rectangular structure as it vibrates due to the suddenly applied earth overpressures. A rectangular mass of earth of thickness equal to half the span of the wall is assumed to move with the wall elements of a rectangular structure.

The roof and exterior wall elements of a buried structure are designed to resist the applied blast loads under the assumption that their edges are rigidly supported against translation. The available data on underground overpressures and underground structural behavior do not justify any closer approximation in the determination of the loading and the structural response. In addition, this simplifies the procedure and will give a reasonable design.

The loads on interior columns and walls, as well as the loads in the plane of the exterior wall and roof elements, are determined as the dynamic reactions of the walls and roof on these elements. The foundation is similarly loaded by the vertical dynamic reactions from the vertical load-carrying elements.

If the buried structure is not anchored firmly in rock, unbalanced lateral forces will tend to cause small lateral movements of the structure relative to the earth which increase the earth pressure on the low pressure side and decrease the pressure on the high pressure side. The net lateral force on buried structures is assumed to be zero, and portions of the structure need not be designed to resist horizontal shears by bending of columns or through shear walls.

In addition to the earth pressures, air- and surface-burst weapons
cause nuclear radiation from which the occupants of a structure must be shielded. The occupants of the structure are shielded by the earth cover and that portion of the structural elements, such as the roof and walls, which lie along a straight line between the interior of the structure and the point of burst. The thickness of the earth cover over buried and semi-buried structures is usually determined by the thickness required to protect the occupants from nuclear radiation.

11-06 EFFECTS OF GROUND MOTION. The detonation of atomic weapons, either underground or near the surface, causes motion in the soil. At equal distances the motion near a detonation is more severe from an underground burst than would be the motion induced by an air-blast pressure acting on the ground surface. Ground motion generally would not harm the structure except if it were located within the crater or zone of rupture. However, ground motion might damage delicate instruments unless they were shock-mounted. The construction of a floor system independent of the main mass of the structure will provide some reduction in shock and motion effects on occupants and floor-mounted equipment.

11-07 DESIGN CRITERIA. The loading to which a buried structure may be subjected from a given weapon cannot be determined with the same degree of accuracy as for aboveground structures. There is also a lack of knowledge of the manner in which a buried structure responds to the loading. Thus, although aboveground structures may be designed for plastic or elastic response, it is advisable to design underground structures to utilize the elastic and elasto-plastic range, but not the fully plastic range of behavior. Where underground structures are to be used for protection of personnel, the factor of safety against collapse thus introduced is justified.

In addition to the structural criteria, a buried structure designed to protect personnel must have sufficient thickness of earth or concrete to protect its occupants from the nuclear radiation given off during the detonation of an atomic weapon. This protection can be obtained by specifying a minimum thickness of earth cover as described in EM 1110-345-413.

11-08 FUNCTIONAL CONSIDERATIONS. A very important problem encountered in
the design of buried structures is that of drainage. If the soil surrounding the structure is well drained, the dynamic earth pressures on the sides of the structure are reduced by 50 percent in the design to resist air-burst effects. The static soil pressure is also reduced. In addition, the interior of the structure is easier to maintain in a dry condition if proper drainage is provided around the structure. It may be necessary to maintain a sump pump to furnish emergency drainage for the structure whether the earth cover or fill is well drained or not. The problem of drainage is minimized for semiburied structures.

Doors for the structure must be designed to resist pressure rebound and suction forces caused by the air blast. The use of doors necessitates some type of ventilation system, unless the structure is to be used for only very brief occupancy. If a ventilation system is provided, the intakes will require mufflers or automatic blast closures to protect the equipment from the air-blast overpressures.

The importance of buried utilities to the proper functioning of a military installation cannot be overemphasized. These buried utilities may include sewer lines, water mains, gas mains, electrical lines, and telephone lines. Connections for utilities should be placed below ground in conduits. Conduits near the surface are stressed by the overpressure induced in the soil by atomic blast.

DESIGN PROCEDURES

11-09 GENERAL. The design procedures for buried and semiburied structures differ with regard to the type of structure, whether rectangular or shell type. Therefore, the design procedure for each type of construction is treated individually.

The design of each element of the structure for the static plus dynamic earth overpressure is preceded by a preliminary design of that element for the static load stresses to which it would normally be subjected. The static design should follow accepted design procedures and specifications. This procedure establishes minimum structural sections for each portion of the structure which may not be decreased in the final design.
The subsequent blast load design may require no change or an increase in the strength of the preliminary sections thus obtained, unless subsequent investigation determines that the structural layout be revised, in which case an additional static design must be undertaken.

11-10 BURIED RECTANGULAR STRUCTURES. The general type of structure discussed here is illustrated in figure 11.3. Typical cross sections are shown to illustrate the structural configuration and general details. Note that the structure lies completely below grade elevation, and the ground surface covering the structure is maintained at its original elevation. The roof is supported by the girders, columns, and walls into which it frames. A mat-type foundation consisting of a slab integral with the walls of the structure or spread footings under the columns and walls may be used to support the structure. The various structural elements are constructed of steel or concrete as required by the type of construction adopted.

The general design procedure for buried rectangular structures to resist the earth overpressures due to air-burst atomic weapons is as follows:

Step 1. Define the design problem. Several items should be considered at this time and values determined which will be used throughout the design. The following items are among those which must be specified.

a. Type of construction material to be used, steel or reinforced concrete.

b. Layout of structure and clearances required.
c. Depth of earth cover to be used.

d. Characteristics of ground in which structure is to be placed including bearing capacity of the soil and drainage problems which may be encountered.

e. Strength of materials to be used in design of structure.

f. Air burst which structure must be designed to resist.

Step 2. Determine the earth overpressure versus time load curves on the roof and sides of the buried structure caused by the air-burst weapon. These load curves can be constructed as outlined in EM 1110-345-413.

Step 3. Design the roof to support the static loading and then investigate this design for its resistance to the static plus dynamic earth overpressures on the roof as computed in step 2. In the dynamic analysis, the mass of the roof element should be increased by the mass of the earth cover. Strengthen the roof section or decrease the roof support spacing as necessary to enable the roof to carry the static plus dynamic loading, and determine the dynamic reaction of the roof on the supporting elements such as purlins, beams, girders, and walls.

Step 4. Design the elements supporting the roof to carry the static loads and investigate their resistance to the static plus dynamic reactions of the roof. Strengthen the supporting element section as necessary to carry the static plus dynamic loading and determine the dynamic reaction of the supporting element. If the roof is supported by purlins or beams which frame into girders, the purlins or beams are designed prior to the girder which must support the dynamic reaction of the purlins or beams. The dynamic reactions of the girder on the columns which support it are finally determined.

Step 5. Design the columns to carry the static loads and investigate their resistance to the static plus dynamic reaction of the girders. Strengthen the column section as necessary to carry the static plus dynamic loading. In buried structures the columns are designed only for a vertical axial load as no net lateral load acts on the structures. The dynamic load factor (D.L.F.) is unity for axial loads on an element so the design of the column for the static plus dynamic reaction of the girders which it supports is essentially a static design problem based upon the ultimate strength of the column under axial load only.
Step 6. Design the exterior walls of the structure to carry the static loads and investigate their resistance to the static plus dynamic earth overpressures on the wall computed in step 2. For the dynamic analysis the mass of the wall element is increased by a rectangular mass of earth of thickness equal to half the span of the wall. Strengthen the wall section as necessary to carry the static plus dynamic loading. The dynamic analysis should include the effect of the dynamic roof, purlin, or beam reactions on the wall resistance. If the exterior columns are built integrally with the walls they must be analyzed for the simultaneous lateral loading on the wall and the vertical reaction from the girders which they support. Determine the dynamic reaction of the wall on its lateral supports.

Step 7. Design the floor to carry the static loads and investigate its resistance to the static plus dynamic loads. Strengthen the floor section as necessary to carry the static plus dynamic loading. If the floor is designed as a mat foundation integral with the rest of the structure, it must carry the vertical and horizontal reactions from the columns and walls as a uniform load distributed over the soil under the floor. However, if the vertical loads from the walls and columns are carried by spread footings, the floor can be designed to carry the horizontal component of the load at the base of the wall as a horizontal strut spanning between the outside walls.

Step 8. Design the footings to carry the static loads into the foundation material and investigate their resistance to the static plus dynamic reactions from the walls and columns. Strengthen the footings as necessary to carry the static plus the dynamic loading.

11-11 SEMIBURIED RECTANGULAR STRUCTURES. The general type of structure discussed here is illustrated in figure 11.4. The structural features and design procedure are similar to those used for buried rectangular structures described in

Figure 11.4. Typical semiburied rectangular structure
paragraph 11-10. The only difference in the design for the two types of construction lies in the determination of the dynamic earth pressure loading when the structure is located in the region of Mach reflection of the air blast. The load computation is discussed under step 2 of the design procedure.

The general design procedure for semiburied rectangular structures to resist the earth overpressures due to air-burst and underground-burst atomic weapons is as follows:

Step 1. Define the design problem. Several items should be considered at this time and values determined which will be used throughout the design. The following items are among those which must be specified.

a. Type of construction material to be used, steel or reinforced concrete.

b. Layout of structure and clearances required.

c. Depth of earth cover to be used.

d. Characteristics of ground in which structure is to be placed including bearing capacity of the soil and drainage problems which may be encountered.

e. Strength of materials to be used in design of structure.

f. Air burst which structure must be designed to resist.

Step 2. Determine the earth overpressure versus time load curves on the roof and sides of the buried structure caused by the air-burst weapon. When the structure is located in the region of regular reflection the air blast is assumed to load the entire surface of the earth covering the structure simultaneously and with the same overpressure everywhere. The earth overpressure load curves for the roof and sides are then determined as outlined in EM 1110-345-413 for structures located in the region of regular reflection.

When the structure is located in the Mach reflection region, the blast approaches the location of the structure from one side and as it strikes the earth cover which slopes up from the ground surface the overpressure on the earth's surface is increased above that in the incident blast wave. The method of obtaining pressures on the surface of the earth cover of a semiburied structure is described in paragraph 3.17 and in
Appendix A of EM 1110-345-413. The transmittal of pressure through the embankment into the structure is a function of the slope of the embankment and density and moisture content of the covering material.

Where the amount of fill is meager, pressure evaluations on a semiburied structure can be made by approximations and charts for the equivalent circle method given in EM 1110-345-413 and by applicable soil mechanics pressure propagation principles.

The same pressure transmittal assumptions can be made for a semiburied structure as are made for a totally buried structure when the horizontal distance from the outside vertical face of the structure to the break in the top of the embankment is equal to or greater than the height of the structure.

Step 3. Design the roof to support the static loading and investigate its resistance to the static plus dynamic earth overpressures on the roof computed in step 2. For the dynamic analysis the mass of the roof element is increased by the mass of the earth cover. Strengthen the roof section or decrease the roof support spacing as necessary to enable the roof to carry the static plus dynamic loading, and determine the dynamic reaction of the roof on the supporting elements such as purlins, beams, girders, and walls.

Step 4. Design the elements supporting the roof to carry the static loads by conventional design methods, and investigate their ultimate resistance to the static plus dynamic reactions of the roof. Strengthen the supporting element section as necessary to carry the static plus dynamic loading and determine the dynamic reaction of the supporting element. If the roof is supported by purlins or beams which frame into girders, the purlins or beams are designed prior to the girder which must support the dynamic reaction of the purlins or beams. The dynamic reactions of the girder on the columns which support it are finally determined.
Step 5. Design the columns to carry the static loads and investigate their resistance to the static plus dynamic reaction of the girders. Strengthen the column section as necessary to carry the static plus dynamic loading. In buried and semiburied structures the columns are designed only for a vertical axial load as no net lateral load acts on the structures. The dynamic load factor (D.L.F.) is unity for axial loads on an element so the design of the column for the static plus dynamic reaction of the girders which it supports is essentially a static design problem based upon the ultimate strength of the column under axial load only.

Step 6. Design the exterior walls of the structure to carry the static loads and investigate their resistance to the static plus dynamic earth overpressures on the wall computed in step 2. For the dynamic analysis the mass of the wall element is increased by a rectangular mass of earth of thickness equal to half the span of the wall. Strengthen the wall section as necessary to carry the static plus dynamic loading. The dynamic analysis should include the effect of the dynamic roof, purlin, or beam reactions on the wall resistance. If the exterior columns are built integrally with the walls they must be analyzed for the simultaneous lateral loading on the wall and the vertical reaction from the girders which they support. Determine the dynamic reaction of the wall on its lateral supports.

Step 7. Design the floor to carry the static loads and investigate its resistance to the static plus dynamic loads. Strengthen the floor section as necessary to carry the static plus dynamic loading. If the floor is designed as a mat foundation integral with the rest of the structure, it must carry the vertical and horizontal reactions from the columns and walls as a uniform load distributed over the soil under the floor. However, if the vertical loads from the walls and columns are carried by spread footings the floor can be designed to carry the horizontal component of the load at the base of the wall as a horizontal strut spanning between the outside walls.

11-12 BURIED SHELL-TYPE STRUCTURES. A typical cross section of the type of structure discussed here is illustrated in figure 11.6. The structure may consist of a barrel or ribbed arch or dome, or a circular section with
a horizontal or vertical axis. The various structural elements are constructed of steel or concrete as required by the type of construction adopted.

The basic difference between the design of shell-type and rectangular structures is the determination of their behavior under the dynamic earth overpressure loading. Arches, domes, and circular elements are loaded practically uniformly throughout by the earth overpressures, and because of their great stiffness under this type of loading the design is based upon a dynamic load factor of unity.

The general design procedure for buried shell-type structures to resist the earth overpressures due to air-burst and underground-burst atomic weapons is as follows:

Step 1. Define the design problem. Several items should be considered at this time and values determined which will be used throughout the design. The following items are among those which must be specified.

a. Type of construction material to be used, steel and/or reinforced concrete.

b. Layout of structure and clearances required.

c. Depth of earth cover to be used.

d. Characteristics of ground in which structure is to be placed including bearing capacity of the soil and moisture conditions.

e. Strength of materials to be used in design of structure.

f. Weapon effects that structure must be designed to resist.

Step 2. Determine the earth overpressure versus time load curves on the arch, dome, or circular element. For arches, domes, and circular sections with a horizontal axis the load curves are constructed as outlined in EM 1110-345-413 for the roof of a buried structure. For circular sections with a vertical axis the load curves are constructed as outlined in EM 1110-345-413 for the sides of a buried structure assuming a zero time of rise. It is assumed for design purposes that the load over the entire
surface of the arch, dome, or circular section is radial and equal to the air-blast overpressure on the ground surface above the structure.

The earth overpressure load curves on plane surfaces bounding the shell surfaces, such as end walls of an arch, are computed as for a similarly located element of a rectangular structure.

Step 3. If the structure is a ribbed arch or dome, design the roof elements spanning between the arch or dome ribs to support the static loading and investigate the resistance to the static plus dynamic overpressures on the roof computed in step 2. For the dynamic analysis the mass of the roof element is increased by the mass of the earth cover. Strengthen the roof section or decrease the spacing of the arch or dome ribs as necessary to carry the static plus dynamic loading, and determine the dynamic reaction of the roof elements on the arch or dome ribs supporting them.

Step 4. Design the main arch, dome, or circular elements to support the static loading. The main supporting elements are the arch or dome ribs, or the curved surfaces of barrel arches, shell-type domes, or uniform circular sections. Investigate the resistance of these elements to the static plus the maximum dynamic load to which they are subjected. The maximum dynamic load should be considered as an additional static load and no dynamic analysis will be involved. It is assumed that the dynamic load is uniformly applied and that the element is very rigid under this type of loading so that a dynamic load factor of unity is used. Strengthen the main supporting elements as necessary to carry the static plus the maximum dynamic load. Determine the maximum reactions developed by the main load-carrying elements if the structure is an arch or dome.

Step 5. Design the columns, walls, or other elements which support the arch or dome to carry the normal static loads. Investigate their resistance to the static plus the dynamic loads. Strengthen these sections as required to carry the static plus the dynamic loads. Determine the dynamic reaction of these elements upon the members which support them.

Step 6. Design the footings to carry the static loads into the foundation material and investigate their resistance to the static plus dynamic reactions from the arch, dome, or circular sections, or from the columns or walls which may support the arch or dome elements.
Strengthen the footings as necessary to carry the static plus the dynamic loading.

11-13 **SEMMBURIED SHELL-TYPE STRUCTURES.** The type of structure discussed here is illustrated in figure 11.7. The structural features are similar to those of the structures discussed in paragraph 11-12. In this case the structure is constructed with a portion of it located above the original grade elevation. The design procedure is similar to that given in paragraph 11-12 for buried shell-type structures. The difference in the design between the two types of construction lies in the determination of the dynamic earth pressure loading and analysis for arch-type structures when located in the Mach stem region of the air blast. Dome-type and circular structures are handled as in the previous paragraph. The load computation is discussed under step 2 of the design procedure.

The general design procedure for semiburied arch, dome, and circular structures to resist the earth overpressures due to air-burst and underground-burst atomic weapons is as follows:

**Step 1. Define the design problem.** Several items should be considered at this time and values determined which will be used throughout the design. The following items are among those which must be specified.

- a. Type of construction material to be used, such as steel or reinforced concrete.
- b. Layout of structure and clearances required.
- c. Depth of earth cover to be used.
- d. Characteristics of ground in which structure is to be placed including bearing capacity of the soil and drainage problems which may be encountered.
- e. Strength of materials to be used in design of structure.
- f. Air burst which structure must be designed to resist.
- g. Underground burst whose effect upon structure is to be investigated.

**Step 2. Determine the pressures on the surface of the earth cover of**
semiburied shell-type structures by methods outlined in EM 1110-345-413 for domes or cylindrical surfaces. Data at the present time are inadequate relative to pressure transmittal through embankments. This transmittal is a function of the slope of the embankment, the thickness of earth through which the pressure is transmitted, and the density and moisture content of the fill. Where the amount of fill is of limited depth, pressure evaluations on a semiburied structure can be made by approximations and charts for the equivalent circle method or by method outlined in Appendix A, EM 1110-345-413, and by application of soil mechanics pressure propagation principles.

The same assumptions can be made for a semiburied shell-type structure as are made for a totally buried structure when the horizontal distance from the springing line of the arch or dome to the top break of the embankment is equal to or greater than the height of the structure.

Step 3. If the structure is a ribbed arch or dome, design the roof elements spanning between the arch or dome ribs to support the static loading and investigate the resistance of the roof section to the static plus dynamic overpressures on the roof computed in step 2. For the dynamic analysis, the mass of the roof element is increased by the mass of the earth cover. Strengthen the roof section or decrease the spacing of the arch or dome ribs as necessary to carry the static plus dynamic loading, and determine the dynamic reaction of the roof elements on the arch or dome ribs supporting them.

Step 4. Design the main arch, dome, or circular elements to support the static loading. The main supporting elements are the arch or dome ribs, or the curved surfaces of barrel arches, shell-type domes, or uniform circular sections. Investigate the resistance of these elements to the static plus the dynamic load to which they are subjected. Strengthen the main supporting elements as necessary, to carry the static plus the dynamic load. Determine the maximum reactions developed by the main load-carrying elements if the structure is an arch or dome. For the case of domes, circular sections, and arches located in the region of regular reflection, the maximum dynamic load can be considered as an additional static load and no dynamic analysis will be involved. It is assumed that the dynamic load
is uniformly applied and that the element is very rigid under this type of loading so that a dynamic load factor of unity is used.

For the case of arch sections located in the Mach stem region of the air blast, the design procedure of EM 1110-345-420 is used. In this situation, the structure is loaded nonsymmetrically and the dynamic behavior must be determined for the nonsymmetrical deflection mode. The mass of the structure for the dynamic analysis includes the mass of the earth cover of a depth not to exceed one-quarter of the arch span. If the arch structure is located in the regular reflection region, it is assumed to be loaded uniformly, and the maximum dynamic load is handled as an additional static load without the necessity of performing a dynamic analysis.

Step 5. Design the columns, walls, or other elements which support the arch or dome or the roof system of cylindrical tank structures to carry the normal static loads. Investigate their resistance to the static plus the dynamic loads. Strengthen these sections as required to carry the static plus the dynamic loads. Determine the dynamic reaction of these elements upon the members which support them.

Step 6. Design the footings to carry the static loads into the foundation material and investigate their resistance to the static plus dynamic reactions from the arch, dome, or circular sections, or from the columns or walls which may support the arch or dome elements. Strengthen the footings as necessary to carry the static plus the dynamic loading.

Step 7. Design the floor to carry the static loads and investigate its resistance to the static plus dynamic loads. Strengthen the floor section as necessary to carry the static plus dynamic loading. If the floor is designed as a mat foundation integral with the rest of the structure, it must carry the vertical and horizontal reactions from the columns and walls as a uniform load distributed over the soil under the floor. However, if the vertical loads from the walls and columns are carried by spread footings, the floor can be designed to carry the horizontal component of the load at the base of the wall as a horizontal strut spanning between the outside walls.
11-14 GENERAL. a. Statement of Problem. The design of a moderate sized rectangular reinforced concrete buried structure is presented as typical of the problems to be encountered in the design of underground structures. The plan and a typical elevation of the structure giving the required minimum clearances are shown in figure 11.8. The roof is a series of two-way slabs supported on girders and columns. The foundation consists of spread footings which support the columns and walls. The lateral thrust at the base of the walls is carried between the walls by the floor slab.

b. Design Procedure. The various design assumptions for this problem are as follows. The structure is located below grade elevation with 10 ft of earth covering the roof. The foundation material is a compact sand-gravel mixture which is drained.

The structure is to be designed for a live load surcharge of 100 psf on the ground surface. This surcharge may be neglected for the dynamic analysis.

The structure is to be designed to resist the air burst of a 20-kt weapon when located within the region of regular reflection with an overpressure of 25 psi on the ground surface. Only elastic and elasto-plastic deflections of the structural elements are to be permitted under the blast loading.

The static design follows procedures recommended in the ACI Building

The material strengths which are used in the design are given below. The dynamic strengths are those recommended in EM 1110-345-414.

Static stresses:

- Steel: $f_s = 20,000$ psi
- $f_y = 40,000$ psi
- Concrete: $f_c' = 3000$ psi, $n = 10$
- Sand-gravel foundation material:
  - Unit weight = 100 lb/ft$^3$
  - Static bearing pressure = 12 kips/ft$^2$

Dynamic stresses:

- Steel: $f_{dy} = 1.3 (40,000) = 52,000$ psi
- Concrete: $f_{dc} = 1.3 (3000) = 3900$ psi
- Sand-gravel foundation material:
  - Ultimate bearing pressure = 30 kips/ft$^2$

11-15 **AIR- AND SURFACE-BURST LOAD CURVES.** The pressure curves on the roof and sides of the structure are determined as outlined in EM 1110-345-413. The structure is located within the region of regular reflection where the shock front strikes the surface of the earth at a sharp angle. Hence, it is assumed that the earth surface above the entire structure is loaded simultaneously by the air blast with a peak overpressure of 25 psi which varies with respect to time as illustrated in figure 11.9. This curve also represents the overpressure versus time curve which is transmitted through the soil to the roof of the buried structure. Since the soil is drained the earth overpressures on the sides of the structure are 50 percent of those on the roof. To determine the average overpressure on the side of the structure a wall height of 12 ft and an average seismic velocity $C_s$ of 3000 fps (see table 3.3) are assumed. This gives a rise time $t_r$ of $12/3000 = 0.004$ sec. The average earth overpressure versus time curve on the sides of the structure is illustrated in figure 11.10.
Figure 11.9. Overpressure versus time curve of air blast on ground surface
Figure 11.10. Average earth overpressure versus time curve on sides of structure

11-16 DESIGN OF ROOF SLAB. a. Static Design Conditions. The roof is designed as a series of square two-way slabs fixed along each edge. The distances center to center of supports are 20 ft by 20 ft, and the clear spans L are assumed to be 18 ft by 18 ft.

Static load: 10 ft of earth cover at 100 lb/ft$^3$ = 1000 lb/ft$^2$
Assume 1-ft-thick slab at 150 lb/ft$^3$ = 150
Live load surcharge at 100 lb/ft$^2$ = 100
Total uniform load on slab w = 1250 lb/ft$^2$

From sec. 709, method 1, of reference [1]

Negative moment at supports = $0.33\left(\frac{1}{11}\right)WL^2 = 0.33\left(\frac{1}{11}\right)1.250(18)^2$
= 12.1 kip-ft/ft

Positive moment = $0.33\left(\frac{1}{16}\right)WL^2 = 0.33\left(\frac{1}{16}\right)1.250(18)^2$
= 8.4 kip-ft/ft

Shear = 0.25 wL = 0.25(1.250)18 = 5.6 kips/ft
b. **Required Slab Properties.**

For $f_s = 20,000$ psi and $f_c = 1350$ psi; $K = 236$ and $a = 1.44$ from reference [2]

$M = Kbd^2$, required $d = \sqrt{\frac{12,100(12)}{236(12)}} = 7.2$ in.

Concrete cover = 2 in. for negative steel and 1 in. for positive steel.

Steel reinforcement in two directions

Use $t = 11$ in., minimum $d_{neg} = 7.5$ in., minimum $d_{pos} = 8.5$ in.

Maximum $v = V/bjd = 5.6(1000)/12(7.5) = 71$ psi < 90 psi allowed

Negative $A_s = M/ad = 12.1/1.44(7.5) = 1.12$ in.$^2$/ft

Use #7 at 6 in., $A_s = 1.20$ in.$^2$/ft, $Eo = 5.5$ in./ft

$p = 1.20/12(7.5) = 0.0133$

$u = V/Eo jd = 5.6(1000)/5.5(7.5) = 155$ psi < 300 psi allowed

Positive $A_s = 8.4/1.44(8.5) = 0.69$ in.$^2$/ft

Use #7 at 10 in., $A_s = 0.72$ in.$^2$/ft, $Eo = 3.3$ in./ft

$p = 0.72/12(8.5) = 0.0071$

Compression steel equivalent to at least 50 percent of the tension steel at a given section is provided in anticipation of requirement for reversed stresses under dynamic loading.

c. **Dynamic Investigation.**

The roof slab section designed in paragraph 11-16a is now investigated to determine its behavior under the dynamic loading illustrated in figure 11.9. The depth of the earth cover is about half of the clear span length of the slab, hence the entire depth of cover is used to increase the mass of the roof. An equivalent resistance curve is determined in order to obtain a rapid check on the dynamic response of the preliminary section obtained through the static design.

d. **Slab Properties.**

Moment of inertia:

At support $p = 0.0133$, $p' = (0.72)/12(7.5) = 0.0080$,

$d = 7.5$ in., $d' = 2.5$ in.
for which \( k = 0.392 \) from table 11 of [2].

\[
I_t = \frac{1}{3} b (kd)^3 + np bd [(1 - k)d]^2 = \frac{1}{3} (12)(0.392)^3(7.5)^3
\]
\[
+ 10(0.0133)(12)(7.5)(1 - 0.392)^2(7.5)^2 = 351 \text{ in.}^4/\text{ft}
\]

\[
I_g = \frac{bt^3}{12} = \frac{1}{12}(12)(11)^3 = 1331 \text{ in.}^4/\text{ft}
\]

At center of slab \( p = 0.0071 \), \( p' = 0.333(1.20)/12(8.5) = 0.0039 \),
\( d = 8.5 \text{ in.}, \ d' = 3.5 \text{ in.} \)

Determine \( k \) for section using table 11 of [2].

\( k = 0.323 \)

\[
I_t = \frac{1}{3} b (kd)^3 + np bd [(1 - k)d]^2 = \frac{1}{3} (12)(0.323)^3(8.5)^3
\]
\[
+ 10(0.0071)(12)(8.5)(1 - 0.323)^2(8.5)^2 = 323 \text{ in.}^4/\text{ft}
\]

\[
I_g = \frac{bt^3}{12} = \frac{1}{12}(12)(11)^3 = 1331 \text{ in.}^4/\text{ft}
\]

\[
I_a = \frac{1}{2} \left[ \frac{1}{2} (I_t + I_g)_{\text{support}} + \frac{1}{2} (I_t + I_g)_{\text{center}} \right]
\]
\[
= 834 \text{ in.}^4/\text{ft}
\]

Determine plastic moment capacity of slab sections using equation (4.18):

\[
M_p = A_s f_{dy} d' + (A_s - A_s') f_{dy} d \left[ 1 - \frac{(A_s - A_s') f_{dy}}{1.7 f_{dc} bd} \right]
\]

At support \( A_s = 1.20 \text{ in.}^2/\text{ft}, A_s' = 0.72 \text{ in.}^2/\text{ft}, \)
\( d = 7.5 \text{ in.}, \ d' = 5 \text{ in.} \)

\[
M_{Ps}^o = 0.72(52)\left(\frac{5}{12}\right) + (1.20 - 0.72)52\left(\frac{7.5}{12}\right) \left[ 1 - \frac{(1.20 - 0.72)52}{1.7(3.9)12(7.5)} \right]
\]
\[
= 30.5 \text{ kip-ft/ft}
\]

At center \( A_s = 0.72 \text{ in.}^2/\text{ft}, A_s' = 0.40 \text{ in.}^2/\text{ft}, \)
\( d = 8.5 \text{ in.}, \ d' = 5 \text{ in.} \)

\[
M_{Ps}^o = 0.40(52)\left(\frac{5}{12}\right) + (0.72 - 0.40)52\left(\frac{8.5}{12}\right) \left[ 1 - \frac{(0.72 - 0.40)52}{1.7(3.9)12(8.5)} \right]
\]
\[
= 20.2 \text{ kip-ft/ft}
\]

Determine net resistance versus deflection curve for the two-way slab using formulas and coefficients from table 6.2b. The static load of 1.15 kips/ft² is used to obtain the net resistance value available to resist the
dynamic loading. The live load surcharge on the earth surface is assumed not to act simultaneously with the air-blast loading.

e. Dynamic Design Factors.

Elastic strain range (from table 6.2b):
\[ R_{lm} = 30.2 \quad M_{Ps}^2 = 30.2(30.5) = 921 \text{ kips} \]
\[ \text{Net } R_{lm} = R_{lm} - \text{ (slab weight)} = 921 - 1.15(18)^2 = 549 \text{ kips} \]
\[ k_l = 870 \quad E I_a/a^2 = 870(3000)834/(18)^2(144) = 46,400 \text{ kips/ft} \]
\[ y_e = \frac{R_{lm}}{k_l} = \frac{549}{46,400} = 0.0118 \text{ ft} \]
\[ K_{LM} = 0.63, \quad V = 0.10 \text{ P} + 0.15 \text{ R} \]

Elasto-plastic strain range (from table 6.2b):
\[ R_m = 24(M_{Pf}^2 + M_{Ps}^2) = 24(30.5 + 20.2) = 1219 \text{ kips} \]
\[ \text{Net } R_m = R_m - \text{ (slab weight)} = 1219 - 1.15(18)^2 = 847 \text{ kips} \]
\[ k_{ep} = 271 \quad E I_a/a^2 = 271(3000)834/(18)^2(144) = 14,600 \text{ kips/ft} \]
\[ y_m = y_e + \frac{R_m - R_{lm}}{k_{ep}} = 0.0118 + \frac{847 - 549}{14,600} = 0.0322 \text{ ft} \]
\[ K_{LM} = 0.67, \quad V = 0.07 \text{ P} + 0.18 \text{ R} \]

The curve of the net available resistance is plotted in figure 11.11. An equivalent elastic resistance curve is computed and used for making a rapid check of the dynamic behavior of the two-way roof slab under the applied load.

f. First Trial - Equivalent Properties. Equivalent elastic curve for elastic and elasto-plastic ranges of strain:

\[ R_{mf} = R_{lm} + R_m \left(1 - \frac{y_e}{y_m}\right) = 549 + 847 \left(1 - \frac{0.0118}{0.0322}\right) = 1085 \text{ kips} \]
\[ k_E = \frac{R_{mp}}{V_m} = \frac{1085}{0.0322} = 33,700 \text{ kips/ft} \]

Average \( K_{LM} = (0.63 + 0.67)/2 = 0.65 \)

Mass of two-way slab and cover = \( m = 1.15(18)^2/32.2 = 11.6 \text{ kip-sec}^2/\text{ft} \)

\[ T_n = 2\pi \sqrt{\frac{mk_{LM}}{k_E}} = 2\pi \sqrt{\frac{11.6(0.65)}{33,700}} = 0.094 \text{ sec} \]

For the type of load curve illustrated in figure 11.9, the time of maximum deflection \( t_m \) lies between 0.25 \( T_n \) and 0.5 \( T_n \) depending upon the relation of \( T_n \) to \( t_o \). For the preliminary check at this point in the design an assumed value of 0.5 \( T_n \) is satisfactory.

Assume \( t_m = 0.5 \ T_n = 0.047 \text{ sec}. \)

Assume a D.L.F. of 2.0 based on the average overpressure between time \( t = 0 \) and \( t = t_m \). From figure 11.9 the average overpressure in this time interval is about 22.5 psi.

Maximum equivalent slab resistance required = \( 2(22.5)(18)^2144/1000 = 2100 \text{ kips} > 1085 \text{ kips available} \).

**g. Revised Slab Properties.** The slab must be strengthened to provide additional resistance. This is done by providing the same amount of positive as negative steel and increasing the depth of the slab to \( 1\frac{1}{4} \text{ in.} \).  

1\(\frac{1}{4}\)-in. slab, positive \( A_s = \) negative \( A_s = 1.20 \text{ in.}^2/\text{ft} \)

Use #7 bars at 6-in. o.c.

Extend half of positive steel through supports and half of negative steel over entire span to provide compression steel equivalent to 50 percent of positive steel for reversed dynamic stresses.

Allow extra 1/2-in. clearance for stirrups

Minimum \( d_{\text{neg}} \) = 10 in., minimum \( d_{\text{pos}} \) = 11 in.

Moment of inertia:

At support \( p = 1.20/10(12) = 0.0100 \), \( p' = 0.5 \), \( p = 0.0050 \),

\[ d = 10 \text{ in.}, \ d' = 3 \text{ in.} \]
for which $k = 0.354$ from table 11 of [2].

$$I_t = \frac{1}{3} b(kd)^3 + np bd [(1 - k)d]^2 + (n - 1)p' bd(kd - 3)^2$$

$$= \left(\frac{1}{3}\right)(12)(0.354)^3(10)^3 + 10(0.0100)12(10)(1.354)^2(10)^2$$

$$+ 9(0.0050)12(10)[(0.354)(10) - 3]^2 = 681 \text{ in.}^4/\text{ft}$$

$$I_g = \frac{bt^3}{12} = (1/12)(12)(14)^3 = 2740 \text{ in.}^4/\text{ft}$$

At midspan $p = 1.20/11(12) = 0.0091, p' = 0.5, p = 0.0045, d = 11 \text{ in.}, d' = 4 \text{ in.}$

for which $k = 0.347$ from table 11 of [2].

$$I_t = \frac{1}{3} b(kd)^3 + np bd [(1 - k)d]^2 = \frac{1}{3}(12)(0.347)^3(11)^3$$

$$+ 10(0.0091)12(11)(1 - 0.347)^2(11)^2 = 842 \text{ in.}^4/\text{ft}$$

$$I_g = \frac{bt^3}{12} = \frac{1}{12}(12)(14)^3 = 2740 \text{ in.}^4/\text{ft}$$

Average $I_a = \frac{1}{2} \left[ \frac{1}{2}(I_t + I_g)_{\text{support}} + \frac{1}{2}(I_t + I_g)_{\text{center}} \right]$ 

$$= 1751 \text{ in.}^4/\text{ft}$$

Plastic moment capacity of slab sections using equation (4.18):

$$M_P = A_s f_{dy} d' + (A_s - A'_s) f_{dy} d \left[ 1 - \frac{(A_s - A'_s) f_{dy}}{1.7 f_{dc} bd} \right] \quad \text{(eq 4.18)}$$

At support $A_s = 1.20 \text{ in.}^2/\text{ft}, A'_s = 0.60 \text{ in.}^2/\text{ft}, d = 10 \text{ in.}, d' = 7 \text{ in.}$

$$M_{Psb} = M_{Psa} = 0.60(52)\left(\frac{7}{12}\right) + (1.20 - 0.60)52\left(\frac{10}{12}\right) \left[ 1 - \frac{0.20 - 0.60}{1.7(3.9)12(10)} \right]$$

$$= 43.2 \text{ kip-ft/ft}$$

At midspan $A_s = 1.20 \text{ in.}^2/\text{ft}, A'_s = 0.60 \text{ in.}^2/\text{ft}, d = 11 \text{ in.}, d' = 7 \text{ in.}$

$$M_{Pfb} = M_{Pfa} = 0.60(52)\left(\frac{7}{12}\right) + (1.20 - 0.60)52\left(\frac{11}{12}\right) \left[ 1 - \frac{0.20 - 0.60}{1.7(3.9)12(11)} \right]$$

$$= 45.8 \text{ kip-ft/ft}$$

h. Revised Slab - Dynamic Design Factors.

Compute net resistance versus deflection curve.
Elastic strain range:

\[ R_{lm} = 30.2 \frac{P_{psb}}{a^2} = 30.2(43.2) = 1307 \text{ kips} \]

Net \[ R_{lm} = R_{lm} - \text{ (slab weight)} = 1307 - 1.15(18)^2 = 935 \text{ kips} \]

\[ k_l = 870 \frac{E I_a}{a^2} = 870(3000)1751/(18)^2144 = 98,000 \text{ kips/ft} \]

\[ y_e = \frac{R_{lm}}{k_l} = 935/98,000 = 0.0095 \text{ ft} \]

\[ K_{LM} = 0.63, V = 0.10 P + 0.15 R \]

Elasto-plastic strain range:

\[ R_m = \frac{1}{a} \left[ 12(M_{Pfa} + M_{Psa}) + 12(M_{Pfb} + M_{Psb}) \right] \]

\[ = \frac{1}{18} \left[ 12(18)(45.8 + 43.2) + 12(18)(45.8 + 43.2) \right] = 2140 \text{ kips} \]

Net \[ R_m = 2140 - 1.15(18)^2 = 1768 \text{ kips} \]

\[ k_{ep} = 271 \frac{E I_a}{a^2} = 271(3000)1751/(18)^2144 = 30,500 \text{ kips/ft} \]

\[ y_m = y_e + \frac{R_m - R_{lm}}{k_{ep}} = 0.0095 + (1768 - 935)/30,500 = 0.0368 \text{ ft} \]

\[ K_{LM} = 0.67, V = 0.07 P + 0.18 R \]

1. Revised Slab - Equivalent Properties. Equivalent elastic curve for elastic and elasto-plastic ranges of strain:

\[ R_{mf} = R_{lm} + R_m \left( 1 - \frac{y_e}{y_m} \right) = 935 + 1768\left( 1 - \frac{0.0095}{0.0368} \right) = 2246 \text{ kips} \]

\[ k_E = \frac{R_{mf}}{y_m} = 2246/0.0368 = 61,100 \text{ kips/ft} \]

Average \( K_{LM} = (0.63 + 0.67)/2 = 0.65 \)

\[ T_n = 2\pi \sqrt{\frac{mK_{LM}}{k_E}} = 2\pi \sqrt{\frac{11.6(0.65)}{61,100}} = 0.070 \text{ sec} \]

Assume \( t_m = 0.5 T_n = 0.035 \text{ sec} \)

Average overpressure between times \( t = 0 \) and \( t = 0.035 \) from figure 11.9 is about 23.5 psi. Assume D.L.F. = 2.0 based on this average overpressure.

Maximum equivalent slab resistance required = \( 2(23.5)(18)^2144/1000 \)

\[ = 2195 \text{ kips} < 2246 \text{ kips available.} \]
j. Determination of **Maximum Deflection and Dynamic Reactions by Numerical Integration**. The 14-in.-slab section appears satisfactory for bending resistance. The final bending analysis is made by a numerical dynamic analysis in which the dynamic reactions are also determined (table 11.1). The time intervals used for the numerical analysis are selected to be about one-tenth of the period in the elastic range of the two-way slab.

| Table 11.1. Determination of Maximum Deflection and Dynamic Reactions for Two-Way Roof Slab |
|---|---|---|---|---|---|---|---|---|
| t (sec) | $P_s$ (psi) | $P_n$ (kips) | $R_n$ (kips) | $P_n - R_n$ (kips) | $\ddot{y}_n (\Delta t)^2$ (ft) | $y_n$ (ft) | $V$ (kips) | Strain Range |
| 0 | 25.0 | 1166 | 0 | 1166/2 | 0.0020 | 0 | 117 | e |
| 0.005 | 24.5 | 1143 | 196 | 947 | 0.0032 | 0.0020 | 144 | e |
| 0.01 | 24.0 | 1120 | 706 | 414 | 0.0014 | 0.0072 | 218 | e |
| 0.015 | 23.6 | 1101 | 1066 | 35 | 0.0001 | 0.0138 | 269 | e-p |
| 0.02 | 23.1 | 1078 | 1271 | -193 | -0.0006 | 0.0205 | 304 | e-p |
| 0.025 | 22.7 | 1059 | 1456 | -397 | -0.0013 | 0.0266 | 336 | e-p |
| 0.03 | 22.2 | 1036 | 1604 | -568 | -0.0018 | 0.0314 | 342 | e-p |
| 0.035 | 21.8 | 1017 | 1695 | -678 | -0.0022 | 0.0344 | 376 | e-p |
| 0.04 | 21.4 | 998 | 1719 | -721 | -0.0023 | 0.0352* | 379** | e-p |
| 0.045 | 20.9 | 975 | 1572 | -597 | -0.0020 | 0.0337 | 334 | e |
| 0.05 | 20.5 | 956 | 1229 | -273 | -0.0009 | 0.0302 | 280 | e |
| 0.055 | 20.1 | 938 | 798 | | | 0.0258 | 214 | e |
| 0.06 | 19.7 | 919 | | | | | |
| 0.065 | 19.3 | 900 | | | | | |

* Maximum deflection $= 0.0352$ ft $< 0.0368$ ft $= y_m$. Therefore 14-in. slab design is satisfactory in bending.

** Maximum shear $= 379$ kips per side.

The elastic range $T_n = \frac{\sqrt{\frac{kn}{k_1} \sqrt{\frac{11.6(0.63)}{98,000}}}}{2\pi} = 0.054$ sec

Use $\Delta t \approx 0.1$ $T_n = 0.005$ sec

By the acceleration impulse extrapolation method (eq 5.49)

$$ y_{n+1} = 2y_n - y_{n-1} + \ddot{y}_n (\Delta t)^2 \quad \text{where} \quad \ddot{y}_n = \frac{(P_n - R_n)}{mK_{LM}} $$

In elastic range $R_n = k_1 y_n = 98,000 y_n$ kips

$$ \ddot{y}_n (\Delta t)^2 = \frac{(P_n - R_n)}{mK_{LM}} (\Delta t)^2 = \frac{(P_n - R_n)(0.005)^2}{11.6(0.63)} = 0.00000342 (P_n - R_n) \text{ ft} $$
V = 0.10 P + 0.15 R
In elasto-plastic range \( R_n = R_{lm} + k_{ep}(y - y_e) = 935 + 30,500(y - 0.0095) \)

\[
y_n(\Delta t)^2 = \frac{(P_n - R_n)(\Delta t)^2}{m_k_{LM}} = (P_n - R_n)(0.005)^2/11.6(0.67)
\]

\[
= 0.00000322(P_n - R_n) \text{ ft}
\]

V = 0.07 P + 0.18 R
If \( y_n < y_{n-1}, R_n = R_{max} - 98,000(y_{max} - y) \)

\[
P_n = (18)^2 144 \text{ p/1000} = 46.6 \text{ P}_s \text{ kips}
\]
\[
P_s \text{ is obtained from figure 11.9.}
\]

k. Shear Strength and Bond Stress.
Design of shear reinforcement:
Maximum dynamic shear = 379 kips per side
Static shear = 0.25(18)^2 1.15 = 93 kips per side
Total shear = (379 + 93)/18 = 26.2 kips/ft
Maximum shear intensity = \( v_{max} = V/bd = 1000(26.2)/12\left(\frac{1}{2}\right)10 \)

\[
= 250 \text{ psi}
\]
Allowable shear stress = \( v_y = 0.04f'_c + 5000 \text{ p} + rf_y \) (eq 4.25a)
Assume \( r = 0, v_{all} = 0.04(3000) + 5000(0.012) \)

\[
= 180 \text{ psi} < 250 \text{ psi actual}
\]
Required \( r = \frac{A_Y}{bs} = \frac{v_{max} - v_y}{f_y} = \frac{250 - 180}{40,000} = 0.0018 \)

\[
A_Y = 0.0018(12)(12) = 0.26 \text{ in.}^2/\text{ft}^2
\]
Use #5 stirrups at 12-in. centers in two directions.
Check bond stress in negative steel at support:
Allowable bond stress given in paragraph 4-09b
\( u = 0.15 f'_c = 0.15(3000) = 450 \text{ psi} \)
Negative steel is #7 bars at 6 in., \( \Sigma o = 5.5 \text{ in.}/\text{ft} \)
Maximum bond stress = \( u_{max} = V/\Sigma o jd = 1000(26.2)/5.5\left(\frac{1}{8}\right)10 \)

\[
= 544 \text{ psi} > 450 \text{ psi allowable}
\]
Try negative steel #6 bars at 4 in., $E_0 = 7.1$ in./ft, $A_s = 1.32$ in./ft

$u_{\text{max}} = 1000(26.2)/7.1(7/8) 10 = 421$ psi < 450 psi allowable

Therefore, use #6 bars at 4 in. for negative steel.

11-17 DESIGN OF ROOF GIRDER. a. Static

Design. Clear span of girders = 18 ft.

Negative moment at supports, assuming complete fixity of girders at each end:

$$M = 0.67\left(\frac{1}{11}\right) w L^2$$

(sec. 709 ref [1])

$$w = \text{load per foot of length}$$

Earth cover $10 \times 100 \times 20 = 20,000$ lb/ft

Surcharge $100 \times 20 = 2,000$

Slab $20 \times \frac{14}{12} \times 150 = 3,500$

Girder (assumed) = 600

26,100 lb/ft

Therefore

$$M = 0.67\left(\frac{1}{11}\right) 26.1(18)^2 = 515 \text{ kip-ft}$$

b. Required Girder Properties.

$M = Kbd^2$, for $f'_c = 3000$ psi, $f_s = 20,000$ psi and $n = 10$, $K = 236(2)$

assuming $b = d/2$

$$d^3 = \frac{M(12)(2)}{K} = \frac{515(12)^2}{0.236}$$

$d = 36.9$ in.

Use $b = 24$ in., $d_{\text{neg}} = d_{\text{pos}} = 35$ in., $t = 38$ in.

Negative steel, at supports:

$$A_s = \frac{M}{f_s jd} = \frac{515(12)}{20(0.88)35} = 10.06 \text{ in.}^2$$

Use 8 #10 bars, $A_s = 10.12 \text{ in.}^2$

Positive steel, at midspan:

$$M = 0.67\left(\frac{1}{16}\right) w L^2$$

(sec. 709, ref [1])

$$= 0.67\left(\frac{1}{16}\right)(26.1)^2(18^2)$$

$$= 354 \text{ kip-ft}$$

$$A_s = \frac{M}{f_s jd} = \frac{354(12)}{20(0.88)35} = 6.9 \text{ in.}^2$$

Use 7 #9 bars, $A_s = 7.00 \text{ in.}^2$
Shear, at support:
\[ V = 0.25 \times W \times L \text{ (sec. 709, ref [1])} \]
\[ = 0.25(26.1)18 = 117 \text{ kips} \]

c. Shear Strength and Bond Stress.
\[ v = \frac{V}{b J d} = \frac{117}{24(0.88)35} = 158 \text{ psi} \]
\[ v_{\text{allowable}} = 0.03 f'_c = 90 \text{ psi} \]

Therefore stirrups required.

Stirrups:

Use #5 bars for stirrups, single loop, \( f_v = 20,000 \text{ psi} \)

Stirrup capacity, \( f_v A_v = 2(20)0.31 = 12.4 \text{ kips} \)

At support, spacing \( S = \frac{f_v A_v}{v'b} = \frac{12.4}{(0.158 - 0.90)24} = 7.6 \text{ in.} \)

Use stirrups spaced as shown in accompanying sketch.

5 at 6 in., 1 at 9 in., 11 at 12 in., 1 at 9 in., 5 at 6 in. over the whole span.

Bond:

Allowable \( u = 0.07 f'_c = 210 \text{ psi} \)

Perimeter, \( \Sigma_0, \text{ required} = \frac{V}{u J d} \)
\[ = \frac{117}{0.210(0.88)35} = 18.1 \text{ in.} \]

Perimeter, \( \Sigma_0, \text{ supplied} = 8 \times 3.99 = 31.9 \text{ in.} \)

Therefore bond requirements satisfied.

d. Dynamic Investigation - Girder Properties.

Moments of inertia:

At midspan - Tee beam action

Effective flange width = 1/2 span = 1/2(18)12 = 108 in. (par. 4-10b)

Gross moment of inertia, \( I_g \)

\[ x = \text{distance to centroid of gross section} \]

Taking static moments about the top of the section

---

31
[108(14) + 24(24)] \bar{x} = 108(14)7 + 24(24)26
\bar{x} = 12.25 \text{ in.}

I_g = \frac{1}{12}(108)14^3 + 108(14)(12.25 - 7.0)^2 + \frac{1}{12}(24)24^3
+ 24(24)(26 - 12.25)^2 = 24,700 + 41,700 + 27,200 + 79,200
= 172,800 \text{ in}^4

Transformed moment of inertia, I_t

Assume all positive steel is extended into the supports, and assume one-half of the negative steel extends over the entire span.

Then A_s = 7.00 \text{ in.}^2, A_s' = 5.06 \text{ in.}^2, d = 35 \text{ in.}, distance to compression steel = 3 \text{ in.}

Assuming the neutral axis lies within the flange of the tee beam

p = \frac{7.00}{108(35)} = 0.00185; p' = \frac{5.06}{108(35)} = 0.00134

for which, from table 11 of [2], k = 0.18

I_t = \frac{1}{3} b(\frac{d}{k})^3 + A_s n[\frac{d}{(1 - k)}]^2 + A_s'(n - 1) [\frac{d}{k} - 3]^2
= \frac{1}{3}(108)(35(0.18))^3 + 7.00(10)(35(0.82))^2
+ 5.06(9)(35(0.18) - 3)^2 = 9000 + 57,600 + 500 = 67,100 \text{ in}^4

I_2 = \frac{1}{2}(I_t + I_g)_{\text{midspan}} = 119,900 \text{ in}^4

At support - rectangular section:

Gross moment of inertia, I_g

I_g = \frac{b d^3}{12} = \frac{1}{12}(24)36^3 = 109,800 \text{ in}^4

Moment of inertia, I_t

A_s = 10.12 \text{ in.}^2, A_s' = 7.00 \text{ in.}^2, d = 35 \text{ in.}, distance to compression steel = 3 \text{ in.}

p = \frac{10.12}{24(35)} = 0.01205, p' = \frac{7.00}{24(35)} = 0.00833

for which, from table 11 of [2], k = 0.34

I_t = \frac{1}{3} b'(kd)^3 + A_s n[(1 - k)(d)]^2 + (n - 1)A_s' [k(d - 3)]^2
= \frac{1}{3}(24)(0.34(35))^3 + 10.12(10)(0.66(35))^2
+ 9(7.00)(0.34(35) - 3)]^2 = 13,500 + 53,100 + 5000
= 71,600 \text{ in}^4

32
\[ I_1 = \frac{1}{2}(I_g + I_t) \text{ support } = 90,700 \text{ in.}^4 \]

**Moment capacities:**

**At support**

\[
M_{Ps} = A_s f_d d' + (A_s - A') f_d y d' \left[ 1 - \frac{(A_s - A') f_d}{1.7 f_d} \right] \quad \text{(eq 4.18)}
\]

\[
= 7.00(52)^{35 - 3} + (10.12 - 7.00)(52)^{35} \left[ 1 - \frac{3.12(52)}{1.7(3.9)(2435)} \right]
\]

\[
= 970 + 460 = 1430 \text{ kip-ft}
\]

**At midspan**

\[
M_{Pm} = 5.06(52)^{35 - 3} + (7.00 - 5.06)(52)^{35} \left[ 1 - \frac{1.94(52)}{1.7(3.9)(10835)} \right]
\]

\[
= 700 + 290 = 990 \text{ kip-ft}
\]

d. **Dynamic Design Factors.**

For \( I_1/I_2 = \frac{90,700}{119,900} = 0.76 \), from figure 6.27, \( f_1 = 10.2 \); from figure 6.29, \( f_3 = 313 \)

**Elastic range:**

- **Net resistance,** \( R_{lm} = M_{Ps} f_1/L - (\text{soil weight above tee beam} + \text{beam weight}) \) table 6.4
  \[
  = 1430(10.2/18) - 18\frac{4(38)}{144}(0.150)
  \]
  \[
  = 2.0(18)(1.0) = 810 - 53 = 757 \text{ kips}
  \]

**Elasto-plastic range:**

- **Net resistance,** \( R_m = \frac{6}{L}(M_{Ps} + M_{Pm}) - (\text{soil weight above tee beam} + \text{beam weight}) \)
  \[
  = \frac{6}{18}(1430 + 990) - 53
  \]
  \[
  = 753 \text{ kips}
  \]

Since \( R_m \approx R_{lm} \), there is no elasto-plastic range

**Resistance diagram:**

(see sketch)

Use \( \frac{1}{2}(I_1 + I_2) = I \) to determine stiffness, \( k_1 \)
\[ k_1 = \frac{fEI}{L^3} = 313(3000)(0.5)(90,700 + 119,900)/144(18)^3 \]
\[ = 112,100 \text{ kips/ft (eq 6.80)} \]

Therefore, \[ y_e = \frac{R_{lm}}{k_1} = 757/112,100 = 0.0067 \text{ ft.} \]

f. **First Trial - Equivalent Properties.**

Mass of tee beam, slab, and cover

\[ m = \frac{1}{32.2} \left[ \frac{1}{2}(18)(20)1.175 + 18(4)0.150 \right] \]
\[ = 6.89 \text{ kip-sec}^2/\text{ft} \]

Natural period of tee beam

\[ T_n = 2\pi\sqrt{\frac{mK_{LM}}{k_1}}, \quad K_{LM} = \text{load mass factor} = 0.830 \text{ from table 6.4} \]

Therefore \[ T_n = 2\pi\sqrt{\frac{6.89(0.830)}{112,100}} = 0.045 \text{ sec} \]

Assume the time to reach maximum displacement \( y_e \) is equal to \( 0.75 \ T_n \), hence

\[ t_{max} = 0.75(0.045) = 0.034 \text{ sec} \]

then, the approximate maximum resistance required,

\[ R_{\text{approx}} = mg + D.L.F. \left[ 2V_1 + 2V_2 \right] \left( \frac{1}{2} \right) \]

where \( V_1 = \text{dynamic slab reaction at time } t = 0 \text{ sec} \)
(\( V_2 = \text{dynamic slab reaction at time } t = 0.034 \text{ sec} \)

D.L.F. = dynamic load factor assumed = 2.0

\[ R_{\text{approx}} = 6.89(32.2) + 2.0[2(117) + 2(376)] \frac{1}{2} \]
\[ = 222 + 986 = 1208 \text{ kips} \]

\[ R_{\text{approx}} = 1208 \text{ kips} > R = 753 \text{ kips} \]

Therefore section inadequate.

g. **Revised Girder Properties.** The tee beam must be strengthened.

Increase positive steel at midspan to 12.48 in.\(^2\), 8 #11 bars. Increase negative steel at supports to 12.48 in.\(^2\), 8 #11 bars. Extend one-half of all positive steel to the supports, and one-half of all negative steel over the entire span length.
Moments of inertia

At midspan, tee beam action:

Gross moment of inertia, \( I_g = 172,800 \text{ in.}^4 \) (from par. 11-17d)

Transformed moment of inertia, assuming neutral axis lies within flange

\[ A_s = 12.48 \text{ in.}^2, \quad A'_s = 6.24 \text{ in.}^2, \quad d = 35 \text{ in.}, \quad \text{distance to compression steel} = 3 \text{ in.}, \quad p = \frac{12.48}{108(35)} = 0.0033, \]

\[ p' = \frac{6.24}{108(35)} = 0.0017 \]

Therefore \( k = 0.22 \) from table 11, ref [2]

\[ I_t = \frac{1}{3} b(kd)^3 + A_s n[d(1-k)]^2 + A'_s(n-1)(kd-3)^2 \]

\[ = \frac{1}{3}(108)[(0.22)(35)]^3 + 12.48(10)[(35)(0.78)]^2 \]

\[ + 6.24(9)[0.22(35) - 3]^2 = 16,400 + 93,300 + 1200 \]

\[ = 110,900 \text{ in.}^4 \]

At support, rectangular section:

Gross moment of inertia, \( I_g = 109,800 \text{ in.}^4 \) (from par. 11-17d)

Transformed moment of inertia

\[ A_s = 12.48 \text{ in.}^2, \quad A'_s = 6.24 \text{ in.}^2, \quad d = 35 \text{ in.}, \quad \text{distance to compression steel} = 3 \text{ in.} \]

\[ p = \frac{12.48}{24(35)} = 0.01484, \quad p' = \frac{6.24}{24(35)} = 0.00742 \]

Therefore \( k = 0.46 \) from table 11 of ref [2]

\[ I_t = \frac{1}{3} b'(kd)^3 + A_s n[(1-k)d]^2 + A'_s(n-1)(kd-3)^2 \]

\[ = \frac{1}{3}(24)[(0.46)(35)]^3 + 12.48(10)[(0.54(35)]^2 \]

\[ + 6.24(9)[(0.46)(35) - 3]^2 = 33,400 + 44,700 + 9600 \]

\[ = 87,700 \text{ in.}^4 \]

At support, \( I_1 = \frac{1}{2}(I_g + I_t) = \frac{1}{2}(109,800 + 87,700) = 98,700 \text{ in.}^4 \)

At midspan, \( I_2 = \frac{1}{2}(I_g + I_t) = \frac{1}{2}(172,800 + 110,900) = 141,800 \text{ in.}^4 \)

\[ \frac{I_1}{I_2} = \frac{98,700}{141,800} = 0.697, \quad \text{from figure 6.27, } f_1 = 10.4; \quad \text{from figure 6.29, } f_3 = 327 \]
Moment capacities

At support:

\[ M_{Ps} = A' \cdot f_{dy} \cdot d' + (A_s - A') \cdot f_{dy} \cdot d \left( 1 - \frac{(A_s - A')}{1.7f_{dc}} \right) \]  
\[ = 6.24(52) \frac{32}{12} + 6.24(52) \frac{35}{12} \left( 1 - \frac{6.24(52)}{1.7(3.9)(24)35} \right) \]  
\[ = 1757 \text{ kip-ft} \]

At midspan:

\[ M_{Pc} = 6.24(52) \frac{32}{12} + 6.24(52) \frac{35}{12} \left( 1 - \frac{6.24(52)}{1.7(3.9)(108)35} \right) \]  
\[ = 1798 \text{ kip-ft} \]

h. Revised Girder - Dynamic Design Factors.

Net resistances (table 6.4):

Elastic range

\[ R_{lm} = M_{Ps} \cdot f_1/L - (\text{soil weight above tee beam + beam weight}) \]  
\[ = \frac{1757(10.4)}{18} - 53 = 962 \text{ kips} \]  
(eq 6.78)

Elasto-plastic range

\[ R_m = \frac{6}{L} (M_{Ps} + M_{Pm}) - (\text{soil weight above tee beam + beam weight}) = \frac{6}{18} (1757 + 1798) - 53 = 1132 \text{ kips} \]

Stiffnesses (table 6.4):

Elastic range

\[ k_1 = EI_1 \cdot f_3/L^3 = 3000(98,700) \cdot 327/144(18)^3 \]  
\[ = 115,200 \text{ kips/ft} \]  
(eq 6.80)

Elasto-plastic range

\[ k_{ep} = 60EI_2/L^3 = 60(3000) \cdot 141,800/144(18)^3 \]  
\[ = 30,400 \text{ kips/ft} \]

Deflections:

Elastic range

\[ y_e = \frac{R_{lm}}{k_1} = \frac{962}{115,200} = 0.00836 \text{ ft} \]

Elasto-plastic range

\[ y_m = \frac{(R_m - R_{lm})}{k_{ep}} + y_e = 170/30,400 + 0.00836 = 0.01394 \text{ ft} \]
i. Revised Girder - Equivalent Properties.

\[ R_{mf} = R_{lm} + R_m \left[ 1 - \frac{y_e}{y_{ep}} \right] \]

\[ = 962 + 1132 \left( 1 - \frac{0.00836}{0.01394} \right) = 1416 \text{ kips} \]

\[ k_E = \frac{R_{mf}}{y_{ep}} = \frac{1416}{0.01394} = 101,600 \text{ kips/ft} \]

Average \( K_{LM} \) for elastic and elasto-plastic ranges = 0.848 (table 6.4)

\[ (T_n)^{\text{equiv}} = 2\pi \sqrt{\frac{mK_{LM}}{k_E}} = 2\pi \sqrt{\frac{6.89(0.848)}{101,600}} = 0.048 \text{ sec} \]

Assume \( t_{\text{max}} = 0.75 \) \( (T_n)^{\text{equiv}} = 0.036 \text{ sec} \)

Approximate resistance required = 1208 kips < \( R_{mf} = 1416 \text{ kips} \)

Therefore section appears satisfactory.

j. Determination of Maximum Deflection and Dynamic Reactions by Numerical Integration.

Use \( \Delta t \approx 0.1 T_n \), therefore \( \Delta t = 0.005 \text{ sec} \)

Elastic range:

\[ \ddot{y} \Delta t^2 = (P_n - R_n) \frac{(\Delta t)^2}{mK_{LM}} = (P_n - R_n) \frac{(0.005)^2}{6.89(0.830)} \]

\[ = 4.372(10^{-6})(P_n - R_n) \]

\( k_l = 115,200 \text{ kips/ft}, y_e = 0.00836 \text{ ft} \)

\( V = 0.44R + 0.06P \) (table 6.4)

Elasto-plastic range:

\[ \ddot{y} \Delta t^2 = (P_n - R_n) \frac{(\Delta t)^2}{mK_{LM}} = (P_n - R_n) \frac{(0.005)^2}{6.89(0.86)} \]

\[ = 4.190(10^{-6})(P_n - R_n) \]

\( k_{ep} = 30,400 \text{ kips/ft}, y_{ep} = 0.01394 \text{ ft} \)

\( V = 0.46R + 0.04P \) (table 6.4)

\( P_n \) = applied load = reactions from slabs = 2 \( V_{\text{slab}} \)
# Table 11.2: Determination of Maximum Deflection and Dynamic Reactions for Roof Girder

<table>
<thead>
<tr>
<th>t (sec)</th>
<th>$V_{slab}$ (kips)</th>
<th>$P_n$ (kips)</th>
<th>$R_n$ (kips)</th>
<th>$P_n - R_n$ (kips)</th>
<th>$y_n^2 \Delta t^2$ (ft)</th>
<th>$y_n$ (ft)</th>
<th>Strain Range</th>
<th>$V_{beam}$ (kips)</th>
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<td>234</td>
<td>---</td>
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</table>

Maximum deflection = 0.009 ft < 0.139 ft, allowable; therefore section satisfactory in bending.

**k. Shear Strength and Bond Stress.**

\[
V_{\text{max}} = \text{maximum dynamic reaction} + \frac{1}{2} \text{total weight of beam and cover} + \frac{1}{2} \text{overpressure on an area equal to width of tee beam stem}
\]

\[
= 481 + \frac{1}{2}(6.89)32 + \frac{1}{2}(2)(18)(22.2)0.144
\]

\[
= 481 + 111 + 56 = 648 \text{ kips}
\]

\[
v_{\text{max}} = \text{shear stress intensity} = \frac{V_{\text{max}}}{bd} = \frac{648(1000)}{24(0.88)35} = 878 \text{ psi}
\]

\[
v_{\text{all}} = 0.04(f_{dc}^') + 5000p = 0.04(3000) + 5000(0.0148) \text{ (eq 4.24b)}
\]

\[
= 194 \text{ psi}
\]

Therefore stirrups required.

Distribution of shear stress throughout girder.

Shear for static load at support which varies along beam consistent with uniform load distribution: \[
\frac{111 + 56}{648}(878) = 226 \text{ psi.}
\]
Shear for dynamic reaction varies along beam consistent with triangular load distribution: 
\[ V = \frac{W}{2L^2} (L^2 - 4x^2) \] 
where \( W = 2 \) reaction, \( x = \) distance from support.

Values for shear intensity at 1-ft increments along span are presented below:

\[ s = \text{spacing} = A_v \left( \frac{f_v}{f_{v'}} \right) \]

Using #6, single-loop stirrups, in pairs

\[ s = 4(0.44)(40,000)/24(v') = 2930/v' \]

Theoretical spacing:

\[ x = 0 \hspace{10pt} 1 \hspace{10pt} 2 \hspace{10pt} 3 \hspace{10pt} 4 \hspace{10pt} 5 \hspace{10pt} 6 \hspace{10pt} 7 \hspace{10pt} 8 \hspace{10pt} 9 \hspace{10pt} \text{ft} \hspace{10pt} \text{ft} \hspace{10pt} \text{ft} \hspace{10pt} \text{ft} \hspace{10pt} \text{ft} \hspace{10pt} \text{ft} \hspace{10pt} \text{ft} \hspace{10pt} \text{ft} \hspace{10pt} \text{ft} \]

\[ s = 4.3 \hspace{10pt} 4.5 \hspace{10pt} 4.8 \hspace{10pt} 5.4 \hspace{10pt} 6.4 \hspace{10pt} 8.2 \hspace{10pt} 10.3 \hspace{10pt} 25.6 \hspace{10pt} \infty \hspace{10pt} \infty \hspace{10pt} \text{in.} \hspace{10pt} \text{in.} \hspace{10pt} \text{in.} \hspace{10pt} \text{in.} \hspace{10pt} \text{in.} \hspace{10pt} \text{in.} \hspace{10pt} \text{in.} \hspace{10pt} \text{in.} \]

\[ \begin{align*}
\text{Bond} & \\
u_{\text{allowable}} &= 0.15 f'_c \quad (3000) \\
&= 450 \text{ psi} \\
\Sigma \text{ required} &= \frac{V}{u_{\text{jd}}} \\
&= 878/0.450(7/8)35 \\
&= 63.5 \text{ in.} \\
\Sigma \text{ provided with 8 #11 bars} &= 35.4 \text{ in.}
\end{align*} \]

Use 18 #9 bars for negative steel,

\[ A_s = 18.00 \text{ in.}^2, \Sigma_o = 63.7 \text{ in.} \]


Maximum \( P \) = weight of concrete slab and beam and weight of soil
\[ \begin{align*}
&= 20(20)\frac{14}{12}(0.150) + 2(20)(4.0)0.150 + 1.1(20)20 \\
&= 70 + 24 + 440 \\
&= 534 \text{ kips}
\end{align*} \]

Try a column section 30 in. by 30 in. with 1 percent steel
Allowable $P = 0.8 \ A' \ (0.225 \ f'_{c} + f'_{s} \ p_{g})$

$= 0.8(30)^{2}[0.225(3) + 20(0.01)]$

$= 630 \text{ kips} > 534 \text{ kips} \text{ required, therefore OK.}$

Use 24 #8 bars and 5/8-in. Ø column ties arranged as shown in section.

b. Dynamic Strength of Column.

Strength required = $4V_{\text{beam}} + P$

$= 4(481) + 534$

$= 1924 + 534 = 2458 \text{ kips}$

$P_{D} = \frac{2A' + f'_{dy}}{2e + 1} + \frac{bt \ f'_{dc}}{3te + 1.178}$ (eq (4.28))

$A' = 7 \#8 \text{ bars} \ 5.54 \text{ in.}^2,$

$f'_{dy} = 52 \text{ psi},$

d = 27.25 in., $d' = 24.5$ in.,

b = 30 in., t = 30 in.,

$e = 0.1 \ d' \text{ (minimum load eccentricity, par. 4.11b)}$

$P_{D} = 470 + 2320$

$= 2790 \text{ kips} > 2458 \text{ kips, OK.}$

11-19 DESIGN OF INTERIOR COLUMN FOOTING. a. Design Conditions.

Assume a compact sand-gravel mixture as the bearing material

Normal bearing pressure = 12 kips/ft$^2$

Ultimate bearing capacity = 30 kips/ft$^2$

Allowable dynamic bearing = 30 kips/ft$^2$

$f'_{c} = 3000 \text{ psi}$

b. Static Design.
Total axial load for an 8- by 8- by 2-ft footing

Column load = 534 kips
Column weight = 2.5(2.5)9(0.150) = 8
Floor weight = 8(8)(1/2) - 6(1/2)(0.150) = 4
Weight of gravel = 8(8) - 6 1/2(0.10) = 3
Weight of footing = 8(8)2(0.150) = 19

Pt = 568 kips

Bearing pressure = 568/8(8)
= 8.87 kips/ft²

Shear and moment:

For a total thickness of footing of 24 in., assume d = 19 in.

c. Shear Strength and Bond Stress.

Check shear at a distance of 19 in. from face of column

Total shear V = (8.87 - 0.42)\{(8)8 - [2.5 + \frac{19}{12}(2)]^2\} = 271 kips
Intensity, v = \frac{V}{bjd} = \frac{271}{4(2.5 + 3.16)12(0.88)19}1000
= 59 psi < 90 psi, therefore OK.

Moment at column face, M = \frac{Wl^2}{2} = 8.45(2.75)2(1/2) = 31.2 kip-ft
Minimum d = \sqrt{\frac{M}{Kb}} = \sqrt{\frac{12(31.200)}{236(12)}} = 11.5 in., OK.

Steel required

A_s = \frac{M}{f_{sjd}} = \frac{31.100(12)}{20,000(0.88)19} (0.85) = 0.95 in.²/ft

Required \Sigma o = 0.85 \frac{V}{u_{jd}} = 0.85[8.42(2.75)1000/300(0.88)19] = 3.92 in.

Use #7 bars 7-in. o.c. each way

\Sigma o = 4.70 in., A_s = 1.03 in.²/ft

d. Dynamic Investigation.

Maximum axial load

Dynamic girder shears (table 11.2) = 4(481) = 1924 kips
Overpressure over girder areas = 2(19)22.2(0.144) = 122
Static load = \frac{568}{2614} kips
\[ P_{\text{soil}} = \frac{P}{A} = \frac{2614}{64} = 40.9 \text{ kips} > 30, \text{ therefore NG.} \]
Area required = \(2614/30 = 87.0 \text{ ft}^2\)

Size = \(\sqrt{87.0} = 9.4\); use 9.5 by 9.5 by 3 ft

Assuming same steel, \#7 bars 7-in. o.c., \(d = 32\) in.

\[ p = \frac{1.03}{32(12)} = 0.0027 \]

Moment capacity, \(M_p = A_s f_{dy} d \left(1 - \frac{p f_{dy}}{1.7 f_{dc}}\right) \quad (\text{eq 4.16}) \]

\[ \begin{align*}
   & = 1.03(52) \frac{32}{12} \left[1 - \frac{0.0027(52)}{1.7(3.9)}\right] \\
   & = 140.0 \text{ kip-ft}
\end{align*} \]

Total moment, \(M\):

Static axial load

Column load + column weight = 542 kips

Weight of 6-in. concrete floor = 6 kips

\[ [9.5(9.5) - 6]0.075 = 6 \]

Weight of gravel = \([9.5(9.5) - 6]\frac{1}{2}(0.10) = 4 \]

Weight of footing = \(9.5(9.5)(3)(0.150) = \frac{41}{593} \]

Dynamic axial load = 2614 - 568

Total = 2639 kips

\[ \begin{align*}
   \text{Weight of footing, floor, and gravel} & = (6 + 4 + 41)/(9.5)^2 \\
   & = 0.57 \text{ kip/ft}^2
\end{align*} \]

Distributed footing reaction

\[ = \frac{2639/(9.5)^2}{29.2 \text{ kips/ft}^2} \]

\[ \text{Bending moment at column face, } M \]

\[ M = \frac{wL^2}{2} = (29.2 - 0.57)(3.5)^2\left(\frac{1}{2}\right)0.85 = 149 \text{ kip-ft} \]

Therefore need more steel.

\[ \text{Approx } A_s \text{ required} = \frac{149}{140}(1.03) = 1.10 \text{ in.}^2/\text{ft} \]

Use \#7 bars 6-in. o.c., \(A_s = 1.20 \text{ in.}^2/\text{ft} \)

Moment capacity, \(M_p\)

\[ p = \frac{1.20}{12(26)} = 0.0038 \]
\[ M_p = A_s f_{dy} \left( 1 - \frac{P_d}{1.7f_{dc}} \right) = 1.20(52) \cdot \frac{32}{12} \left( 1 - \frac{(0.0038)52}{1.7(3.9)} \right) \text{ (eq 4.16)} \]

= 162 kip-ft > 149 kip-ft, therefore OK.

Shear at 32 in. from column face
\[ V = (29.2 - 0.57)[9.5(9.5) - (7.83)7.83] = 824 \text{ kips} \]
\[ v = \frac{V}{4bjd} = \frac{824/4(7.83)(12)(\frac{1}{8})}{32}1000 = 78 \text{ psi} \]
\[ v = 0.04f'_c + 5000p \text{ (eq 4.24b)} \]
Allowable \[ v = 0.04(3000) + 5000(0.0038) \]
= 120 + 19 = 139 psi > 78 psi, therefore OK.

Bond at face of column
Allowable \[ u = 450 \text{ psi} = 0.15f'_d \]
\[ \Sigma_0 \text{ required} = 0.85 \cdot \frac{V}{u,jd} = 0.85[28.6(3.5)/0.450(0.88)32] \]
= 6.71 in./ft
Therefore use #6 bars \(\frac{3}{4}\)-in. o.c.
\[ \Sigma_0 = 7.1 \text{ in., } A_s = 1.32 \text{ in.}^2/\text{ft} \]

11-20 DESIGN OF WALL. a. Static Design.

Computation of fixed end moments:
Lateral at rest pressure \(\approx 0.5\) hydrostatic pressure
\[ P_B = 0.5[11(0.1) + 0.1] = 0.60 \text{ kip/ft}^2 \]
\[ P_A = 0.5[21.5(0.1) + 0.1] = 1.125 \text{ kips/ft}^2 \]
Fixed end moments (considering a 1-ft-wide strip):

\[
\text{FEM}_{BA} = \frac{w_B L^2}{12} + \frac{1}{15} \left( \frac{w_A - w_B}{2} \right)^2
\]

\[
= \frac{1}{12} (0.600) 10.5^2 + \frac{1}{15} (1.125 - 0.600) \left( \frac{1}{2} \right) 10.5^2
\]

\[
= 5.52 + 1.93 = 7.45 \text{ kip-ft}
\]

\[
\text{FEM}_{AB} = \frac{1}{12} w_B L^2 + \frac{1}{10} \left( \frac{w_A - w_B}{2} \right)^2
\]

\[
= \frac{1}{12} (0.600) 10.5^2 + \frac{1}{10} (1.125 - 0.600) \left( \frac{1}{2} \right) 10.5^2
\]

\[
= 5.52 + 2.89 = 8.41 \text{ kip-ft}
\]

Assuming the wall to be fixed at the top and pinned at the base, then the maximum negative moment, at the top of the wall, \( M_B = \text{FEM}_{BA} + (C.O.) \text{FEM}_{AB} \)

\[
M_B = 7.45 + 0.5 (8.41) = 11.65 \text{ kip-ft}
\]

b. Required Wall Properties. The negative moment at the top of the wall will control the required depth. Since this moment \( M_B \) is less than that required by the static roof design the wall will be made the same thickness as the roof. In order to develop full plastic action at the top of the wall, the wall and the roof should have approximately the same amount of reinforcement. Therefore, the wall will be made 14 in. thick and reinforced with negative steel of #7 bars 6 in. center to center, \( A_s = 1.20 \text{ in.}^2/\text{ft} \). Extend one-half of all negative steel over the entire height of the wall.

Positive steel:

Positive moment

\[
R_B = \frac{M_B}{L} + \frac{1}{2} w_B L + \frac{1}{3} (w_A - w_B) \frac{L}{2} = \frac{11.65}{10.5} + \frac{1}{2} (0.600) 10.5
\]

\[
+ \frac{1}{3} (0.525) \left( \frac{10.5}{2} \right) = 1.11 + 3.15 + 1.05 = 5.31 \text{ kips}
\]

\[
R_A = - \frac{M_B}{L} + \frac{1}{2} w_B L + \frac{2}{3} (w_A - w_B) \frac{L}{2} = - \frac{11.65}{10.5} + \frac{1}{2} (0.600) 10.5
\]

\[
+ \frac{2}{3} (0.525) \left( \frac{10.5}{2} \right) = -1.11 + 3.15 + 2.10 = 4.14 \text{ kips}
\]
Maximum positive moment occurs a distance \( x \) above the base of the wall at the point of zero shear. The point of zero shear is given by
\[
4.14 - 1.125x + 0.05x^2/2 = 0
\]
x = 4.04 ft

Moment, \( M_{\text{pos}} = 4.14(4.04) - 1.125\left(\frac{4.04}{2}\right) + 4.04(0.05)\left(\frac{4.04}{2}\right) \)
= 16.72 - 9.18 + 0.55
= 8.09 kip-ft

then
\[
A_s = \frac{M}{f_{s}d}, \text{ assuming } d = 10 \text{ in.}
\]
\[
A_s = \frac{8.09(12)}{20(0.88)10} = 0.55 \text{ in.}^2
\]
Therefore use #7 bars 12 in. center to center and extend all bars over the entire wall span, \( \Sigma_o = 2.75 \text{ in.} \)

c. Shear Strength and Bond Stress.
\[
V = 5.31 \text{ kips at top}
\]
\[
v = \frac{V}{bjd} = \frac{5310}{12(0.88)10} = 50 \text{ psi} < 90 \text{ psi, OK.}
\]
\[
V = 4.14 \text{ kips at base}
\]
\[
u = \frac{V}{\Sigma ojd} = \frac{4140}{2.75(0.88)10} = 171 \text{ psi} < 300 \text{ psi, OK.}
\]
d. Dynamic Investigation.
Determination of blast loads on wall:
Assume ground shock velocity, \( c_s = 3000 \) fps, with ground shock traveling vertically downward, \( t_r = \text{rise time} = 10.5/3000 = 0.0035 \text{ sec.} \) Since \( t_r \) is quite small, assume that \( t_r = 0 \) in the computation of the overpressure vs time curve for the wall. Therefore, the same overpressure-time curve (figure 11.9) used in the design of the roof slab may be used. And, \( P \), the load on the wall
\[
P = 0.5 I_{\text{wall}} P_s = 0.5(10.5)0.144 P_s
\]
\[
= 0.755 P_s
\]
e. First Trial - Wall Properties.
Moments of inertia at midspan, considering a 1-ft-wide strip
Gross moment of inertia

$$I_g = \frac{bt^3}{12} = \frac{1}{12}(12)14^3 = 2740 \text{ in.}^4$$

Transformed moment of inertia

$$A_s = A_s' = 0.60 \text{ in.}^2/\text{ft}, \quad d = 11 \text{ in.}, \quad \text{distance to compression steel} = 4 \text{ in.}$$

$$p = p' = \frac{0.60}{12(11)} = 0.0044$$

Therefore \( k = 0.28 \) from table 11 of reference [2]

$$I_t = \frac{1}{3}b(kd)^3 + A_s n[(1 - k)d]^2 = \frac{1}{3}(12)[0.28(11)]^3$$
$$+ 0.60(10)[0.72(11)]^2 = 117 + 377 = 494 \text{ in.}^4$$

Average moment of inertia

$$I_a = \frac{1}{2}(I_g + I_t) = \frac{1}{2}(2740 + 494) = 1617 \text{ in.}^4$$

Moment capacities (considering a strip 1 ft wide):

At fixed support (top of wall)

$$A_s = 1.20 \text{ in.}^2, \quad A_s' = 0.60 \text{ in.}^2, \quad d' = 7 \text{ in.}, \quad d = 10 \text{ in.}$$

$$M_{ps} = A_s' d' f_{dy} + (A_s - A_s')f_{dy} \frac{d}{1.7f_{dc} bd} \left(1 - (A_s - A_s') \frac{f_{dy}}{1.7f_{dc} bd}\right)$$

(eq 4.18)

$$= 0.60(\frac{7}{12})52 + 0.60(52)\frac{10}{12} [1 - 0.60(52)/1.7(3.9)(12)10]$$

$$= 18.2 + 25.0 = 43.2 \text{ kip-ft}$$

At midspan

$$A_s = A_s' = 0.60 \text{ in.}^2, \quad d' = 7 \text{ in.}, \quad d = 11 \text{ in.}$$

$$M_{pc} = A_s' d' f_{dy} = 0.60(\frac{7}{12})52 = 18.2 \text{ kip-ft}$$

f. Dynamic Design Factors. Table 6.1c (considering a 1-ft strip):

Elastic range:

Net resistance \( R_{lm} = \frac{8M_{ps}}{L} - \text{(lateral earth load)} \)

$$= \frac{8(43.2)}{10.5} - (0.600 + 1.125)(\frac{1}{2})10.5$$

$$= 32.9 - 9.1 = 23.8 \text{ kips}$$

Stiffness \( k_l = \frac{185EI}{L^3} = 185(3000)1617/10.5^3(144) \)

$$= 5380 \text{ kips/ft}$$

Deflection \( y_e = \frac{R_{lm}}{k_l} = 23.8/5380 \)

$$= 0.00443 \text{ ft}$$
Elasto-plastic range:

Net resistance \( R_m = \frac{h}{L} (M_{Ps} + 2M_{Pc}) - \) (lateral earth load)

\[ = \frac{h}{10.5} \ [43.2 + 2(18.2)] - 9.1 \]

\[ = 21.3 \text{ kips} < R_{lm} = 23.8 \text{ kips} \]

Since \( R_m < R_{lm} \), there is no elasto-plastic range.

Revised \( y_e = R_m/k_1 = 21.3/5380 \)

\[ = 0.00396 \text{ ft} \]

Mass of wall strip, 1 ft wide

Assume the mass of the wall to be increased by an effective depth of soil equal to one-half the span of the wall.

\[ \text{Mass, } m = \frac{1}{32.2} \left[ 14(12) \left( \frac{1}{144} \right) (10.5)0.150 + 10.5 \left( \frac{10.5}{2} \right)0.1 \right] \]

\[ = 0.2285 \text{ kip-sec}^2/\text{ft} \]

g. First Trial - Equivalent Properties.

Investigate use of elastic range only of resistance diagram.

\( R_m = 21.3 \text{ kips} \)

\( k_1 = 5380 \text{ kips/ft} \)

\( K_{LM} \) for elastic range = 0.78 (table 6.1c)

\[ (T_n)_{equiv} = 2\pi \sqrt{\frac{m K_{LM}}{k_1}} = 2\pi \sqrt{\frac{0.2285(0.78)}{5380}} = 0.0362 \text{ sec} \]

Assume \( t_{max} = 0.5(T_n)_{equiv} = 0.018 \text{ sec} \)

Average load \( 0 < t < 0.018 \) is about \( \frac{1}{2}(25.0 + 23.4) = 24.2 \text{ psi} \)

Approximate elastic resistance required = \( 2(24.2)(0.755) \)

\[ = 36.5 \text{ kips} > R_m = 21.3 \text{ kips} \]

It appears that the wall is not strong enough for only elastic action without vertical loads. Therefore the strengthening effect of the axial loads will be investigated before performing the dynamic analysis.

h. Effect of Axial Load on Wall.

P_D versus M_D curve at top of wall
b = 12 in., d = 10 in., t = 14 in.

\[ p = \frac{1.20}{10(12)} = 0.010, \quad p' = 0.005 \]

Tension steel failure

\[ P_D = 0.85 f'_{dc} \frac{bt}{t} \]

\[ \left\{ \sqrt{\frac{[e/t - 0.5 + (p - p')m]^2 + \frac{2d'p'm}{t}}{t} + (p - p')m \left[ \frac{2d}{t} - (p - p')m \right]} \right\} \]

\[ d' = 7\text{-in. distance between tension and compression steel} \]

\[ f'_{dc} = 3900 \text{ psi}, \quad f'_{dy} = 52,000 \text{ psi} \]

\[ m = f'_{dy} / 0.85 f'_{dc} = 15.69, \quad (p - p')m = 0.0785 \]

\[ P_D = 0.85(3.9)(12)14 \]

\[ \left\{ \sqrt{\frac{[e/t - 0.5 + 0.0785]^2 + \frac{2(7)(0.005)15.69}{14} + 0.0785\left[ \frac{20}{14} - 0.0785 \right]}{t} - [e/t - 0.5 + 0.0785]} \right\} \]

\[ = 557 \left\{ \sqrt{[e/t - 0.4215]^2 + 0.1847 - [e/t - 0.4215]} \right\} \]

Table 11.3. Tabulation of Ultimate Axial Load vs Ultimate Moment (Top of Wall)

<table>
<thead>
<tr>
<th>e/t</th>
<th>P_D (kips)</th>
<th>e (in.)</th>
<th>M_D (ft-kips)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>79.0</td>
<td>14</td>
<td>92.2</td>
</tr>
<tr>
<td>1.5</td>
<td>50.5</td>
<td>21</td>
<td>88.5</td>
</tr>
<tr>
<td>4</td>
<td>14.3</td>
<td>56</td>
<td>66.7</td>
</tr>
<tr>
<td>5</td>
<td>11.2</td>
<td>70</td>
<td>65.4</td>
</tr>
<tr>
<td>6</td>
<td>9.2</td>
<td>84</td>
<td>64.4</td>
</tr>
<tr>
<td>8</td>
<td>6.8</td>
<td>112</td>
<td>63.4</td>
</tr>
<tr>
<td>10</td>
<td>5.4</td>
<td>140</td>
<td>63.0</td>
</tr>
</tbody>
</table>

\[ P_D \text{ versus } M_D \text{ curve at midheight of wall.} \]

\[ b = 12 \text{ in., } d = 11 \text{ in., } t = 14 \text{ in.} \]

\[ p = \frac{0.60}{11(12)} = p' = 0.00455 \]

\[ (p - p')m = 0 \]
\[ P_D = 557 \left\{ \sqrt{[e/t - 0.5]^2 + \frac{2(7)(0.00455)15.69}{14}} - [e/t - 0.5] \right\} \]
\[ = 557 \left\{ \sqrt{[e/t - 0.5]^2 + 0.071373} - [e/t - 0.5] \right\} \]

Table 11.4. Tabulation of Ultimate Axial Load vs Ultimate Moment (Midheight of Wall)

<table>
<thead>
<tr>
<th>e/t</th>
<th>( P_D ) (kips)</th>
<th>e (in.)</th>
<th>( M_D ) (ft-kips)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>37.3</td>
<td>14</td>
<td>43.5</td>
</tr>
<tr>
<td>1.5</td>
<td>18.6</td>
<td>21</td>
<td>32.6</td>
</tr>
<tr>
<td>2</td>
<td>13.1</td>
<td>28</td>
<td>30.6</td>
</tr>
<tr>
<td>3</td>
<td>7.9</td>
<td>42</td>
<td>27.6</td>
</tr>
<tr>
<td>4</td>
<td>5.7</td>
<td>56</td>
<td>26.6</td>
</tr>
<tr>
<td>5</td>
<td>4.4</td>
<td>70</td>
<td>25.7</td>
</tr>
<tr>
<td>6</td>
<td>3.6</td>
<td>84</td>
<td>25.2</td>
</tr>
<tr>
<td>8</td>
<td>2.6</td>
<td>112</td>
<td>24.3</td>
</tr>
<tr>
<td>10</td>
<td>2.1</td>
<td>140</td>
<td>24.2</td>
</tr>
</tbody>
</table>

1. Determination of Maximum Deflection and Dynamic Reactions by Numerical Analysis. A dynamic analysis is now performed to determine the dynamic behavior of the wall under the blast loading. Revised values of \( R_{lm} \) and \( R_m \) are determined as a function of the axial load \( P_D \). \( P_D \) versus \( M_D \) tables are used to obtain \( M_D \) at the top and midheight of the wall from which the corresponding values of \( R_{lm} \) and \( R_m \) are computed using equations from table 6.16. The axial load per foot of wall \( P_D \) is obtained from the dynamic and static roof reactions (par. 11-17d and par. 11-17j).

Use \( \Delta t \approx 0.1 T_n \), \( \Delta t = 0.005 \) sec

Elastic range:

\[ \ddot{y} \Delta t^2 = \left( \frac{P_n - R_n}{mK_{LM}} \right) \frac{(\Delta t)^2}{(0.2285)0.78} \]
\[ = 1.400(10)^{-4} (P_n - R_n) \]
\[ k_1 = 5380 \text{ kips/ft} \]
\[ V_B = 0.43 \, R_n + 0.19 \, P_n \text{ (table 6.1c)} \]
\[ V_A = 0.26 \, R_n + 0.12 \, P_n \]
\[ P_n = 0.755 \, P_s \text{ (par. 6.21b)} \]
\[ R_n = k_1 \times n \]

**Table 11.5. Determination of Maximum Deflection and Dynamic Reactions for Wall Slab (Including Effects of Axial Load)**

<table>
<thead>
<tr>
<th>t (sec)</th>
<th>Axial Load (kips)</th>
<th>( R_{lm} ) (kips)</th>
<th>( R_s ) (kips)</th>
<th>( P_s ) (psi)</th>
<th>( P_n ) (kips)</th>
<th>( R_m ) (kips)</th>
<th>( P_n - R_{lm} ) (kips)</th>
<th>( Y_n (at)^2 ) (ft)</th>
<th>( Y_n ) (ft)</th>
<th>Strain Range</th>
<th>( V_B ) (kips)</th>
<th>( V_A ) (kips)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>12</td>
<td>40.4</td>
<td>48.0</td>
<td>25.0</td>
<td>18.9</td>
<td>0</td>
<td>18.9/2</td>
<td>0.0013</td>
<td>0</td>
<td>e</td>
<td>3.6</td>
<td>2.3</td>
</tr>
<tr>
<td>0.005</td>
<td>13</td>
<td>41.6</td>
<td>48.5</td>
<td>24.5</td>
<td>18.5</td>
<td>7.0</td>
<td>11.5</td>
<td>0.0016</td>
<td>0.0013</td>
<td>e</td>
<td>6.5</td>
<td>4.0</td>
</tr>
<tr>
<td>0.010</td>
<td>17</td>
<td>43.4</td>
<td>50.1</td>
<td>24.0</td>
<td>18.1</td>
<td>22.6</td>
<td>-4.5</td>
<td>-0.0006</td>
<td>0.0042</td>
<td>e</td>
<td>13.1</td>
<td>8.1</td>
</tr>
<tr>
<td>0.015</td>
<td>20</td>
<td>45.4</td>
<td>50.9</td>
<td>23.6</td>
<td>17.8</td>
<td>35.0</td>
<td>-17.2</td>
<td>-0.0024</td>
<td>0.0065</td>
<td>e</td>
<td>18.5</td>
<td>11.2</td>
</tr>
<tr>
<td>0.020</td>
<td>22</td>
<td>46.5</td>
<td>51.2</td>
<td>23.1</td>
<td>17.4</td>
<td>34.4</td>
<td>0.0064</td>
<td>e</td>
<td>18.1</td>
<td>11.0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Note: The wall dynamic behavior is entirely elastic; therefore the section is adequate in bending.*

**j. Shear Strength and Bond Stress.**

At top of wall:

Maximum shear = \( V_{dyn} + V_{static} = 18.5 + 5.3 = 23.8 \text{ kips} \)

\[
V' = \frac{V}{b d} = \frac{23,800}{12(0.88)10} = 225 \text{ psi}
\]

\[
V_{all} = 0.04f_c' + 5000p \quad (\text{eq } 4.24b)
\]

\[
= 120 + 50 = 170 \text{ psi}
\]

Using 1/2-in. \( \phi \) stirrups \( A_v = 0.20 \text{ in.}^2 \)

\[
s = \text{spacing} = f_v A_v/bv' = \frac{40,000(0.20)}{12(225 - 170)} = 12.1 \text{ in.}
\]

At base of wall:

\[
V = 11.2 + 4.1 = 15.3 \text{ kips}
\]

\[
V' = \frac{V}{b d} = 15,300/12(0.88)11 = 144 \text{ psi}, \text{ no stirrups required}
\]

Use 1/2-in. \( \phi \) single-loop stirrups, one rod per 12 in. of wall length, 12-in. o.c. over entire height

Bond, \( u_{allowable} = 450 \text{ psi} \)

At top of wall:

\[
\Sigma \text{ required} = \frac{V}{u_{jd}} = 23,800/450(0.88)10 = 6.01 \text{ in.}
\]
\[ \Sigma_0 \text{ provided} = 2 \times 2.75 = 5.5 \text{ in.}, \quad 5.5 < 6.01, \text{ therefore NG.} \]

Instead of \#7 bars 6-in. o.c. for negative steel use \#7 bars 5\(\frac{1}{2}\) in. o.c.

\[ A_s = 1.31 \text{ in.}^2, \quad \Sigma_0 = 6.0 \text{ in.} \]

At base of wall:

\[ \Sigma_0 \text{ required} = \frac{V}{u \times d} = \frac{15,300/450(0.88)11}{2.75} = 3.50 \text{ in.} \]

\[ \Sigma_0 \text{ provided} = 2.75 \text{ in.}, \quad 2.75 < 3.50, \text{ therefore NG.} \]

Instead of \#7 bars 12-in. o.c. for positive steel use \#5 bars 5\(\frac{1}{2}\) in. o.c.

\[ A_s = 0.68 \text{ in.}^2, \quad \Sigma_0 = 4.30 \text{ in.} \]

11-21 **DESIGN OF EXTERIOR WALL**

**COLUMN.** Assume a column section 2 ft by 2 ft.

a. **Design Conditions.** The exterior columns will frame into spread footings at the base. Therefore, the column will be analyzed as being fixed at top and bottom.

The negative steel in the column, at top and bottom, shall be taken equal to the negative steel in the roof girder. Use eight \#11 bars, \(A'_s = 12.48 \text{ in.}^2\). Extend one-half of the negative steel over the entire height of the wall. Take the positive steel at midspan equal to one-half the negative steel; i.e., \(A_s = 6.24 \text{ in.}^2 = 4 \#11 \text{ bars. All positive steel will be extended over the entire wall height.} \)

Tee beam action will be taken into account.

b. **Column Properties.**

Moments of inertia:

Support:

\[ G = \frac{1}{12} b t^3 = \frac{1}{12}(24)(24)^3 = 27,650 \text{ in.}^4 \]

Transformed take \(d = 21 \text{ in.},\ d' = 3 \text{ in.},\ p = \frac{12.48}{24(21)} = 0.0248\)

\[ p' = \frac{6.24}{24(21)} = 0.0124 \]

Therefore \(k = 0.45 \)

\[ 51 \]
\[ I_t = \frac{1}{3} b(kd)^3 + (n - 1) A_s' (kd - 4)^2 + nA_s [(1 - k)d]^2 \]
\[ = \frac{1}{3} (24)[0.45(21)]^3 + 9(6.24)[0.45(21) - 4]^2 + 10(6.24)[0.74(21)]^2 \]
\[ = 3420 + 120 + 15,030 \]
\[ = 18,570 \text{ in.}^4 \]

\[ I_l = \frac{1}{2} (I_t + I_g) = \frac{1}{2} (25,680 + 27,650) \]
\[ = 26,600 \text{ in.}^4 \]

**Midspan:**

Effective flange width = \( \frac{1}{2} \) beam span = \( \frac{1}{2} \) (10.5) ft = 63 in.

**Gross I**

\[ [63(14) + 10(24)] \overline{x} = 63(14)7 + 24(10)19 \]
\[ \overline{x} = 5.5 + 4.1 = 9.6 \text{ in.} \]

\[ I_g = \frac{1}{12} (63) 14^3 + 63(14) 2.6^2 + \frac{1}{12} (24) 10^3 + 24(10) 9.4^2 \]
\[ = 43,580 \text{ in.}^4 \]

**Transformed, assume neutral axis lies within flange:**

\[ A_s = A_s' = 6.24 \text{ in.}^2, \quad p = p' = \frac{6.24}{63(21)} = 0.00472 \]

Therefore \( k = 0.26 \)

\[ I_t = \frac{1}{3} b(kd)^3 + (n - 1) A_s' (kd - 4)^2 + nA_s [(1 - k)d]^2 \]
\[ = \frac{1}{3} (63)[0.26(21)]^3 + 9(6.24)[0.26(21) - 4]^2 + 10(6.24)[0.74(21)]^2 \]
\[ = 3420 + 120 + 15,030 \]
\[ = 18,570 \text{ in.}^4 \]

\[ I_2 = \frac{1}{2} (I_g + I_t) = \frac{1}{2}(43,580 + 18,570) = 31,080 \text{ in.}^4 \]
\[ I_1/I_2 = 0.853 \]

Therefore \( f_1 = 9.93 \),

\( f_2 = 0.656 \)

\( f_3 = 294 \) (table 6.4)
Moment capacity:

\[ M_{Ps} = A'_s f_{dy} (d') + (A_s - A'_s) f_{dy} d \left[ 1 - \frac{f_{dy}}{1.7 f'_{dc} b d} \right] \]  
\[ = 6.24(52)17/12 + 6.24(52) \frac{20}{12} \left[ 1 - \frac{6.24(52)}{24(20)(1.7)3.9} \right] \]
\[ = 460 + 486 = 946 \text{ kip-ft} \]

\[ M_{Pe} = A'_s f_{dy} (d') \] (eq 4.19)
\[ = 6.24(52)17/12 = 460 \text{ kip-ft} \]

C. Axial Load - Moment Interaction.

Tee section tension steel failure:

\[ A_s = A'_s \]
\[ \bar{X} = \text{depth to centroid} \]
\[ a = \text{depth of stressed concrete} \]
\[ P = \text{eccentric axial load} \]
\[ e = \text{eccentricity} \]
\[ P = 0.85 f'_{dc} ba \]

Therefore \[ a = \frac{P}{0.85 f'_{dc}} \cdot b = \frac{P}{0.85(3.9)63} = 0.00479 P \]

\[ M = Pe = 0.85 f'_{dc} ba(\bar{X} - a/2) + A_s f_{dy} d' \]
\[ = 0.85(3.9) \frac{63}{12} a (9.6 - a/2) + 6.24(52)17/12 \]
\[ = 17.4 a (9.6 - a/2) + 460 \]

Table 11.6. Tabulation of Ultimate Axial Load vs Ultimate Moment (Column-Tee Section at Midheight of Wall)

<table>
<thead>
<tr>
<th>P (kips)</th>
<th>a (in.)</th>
<th>M (ft-kips)</th>
<th>P</th>
<th>a</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>460</td>
<td>300</td>
<td>1.44</td>
<td>683</td>
</tr>
<tr>
<td>50</td>
<td>0.24</td>
<td>500</td>
<td>350</td>
<td>1.68</td>
<td>716</td>
</tr>
<tr>
<td>100</td>
<td>0.48</td>
<td>538</td>
<td>400</td>
<td>1.92</td>
<td>749</td>
</tr>
<tr>
<td>150</td>
<td>0.72</td>
<td>576</td>
<td>450</td>
<td>2.16</td>
<td>780</td>
</tr>
<tr>
<td>200</td>
<td>0.96</td>
<td>612</td>
<td>500</td>
<td>2.40</td>
<td>811</td>
</tr>
<tr>
<td>250</td>
<td>1.20</td>
<td>648</td>
<td>750</td>
<td>3.59</td>
<td>948</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1000</td>
<td>4.79</td>
<td>1060</td>
</tr>
</tbody>
</table>
Compression failure:

Axial load capacity, \( e = 0 \)

\[
M = 0, \ P = A_c 0.85f'_{dc} + (A_s + A_s')f_{dy} = [63(14) + 10(24) - (A_s + A_s')] \times (0.85)3.9 + 12.48(52) = 3680 + 650 = 4330 \text{ kips}
\]

Moment capacity \( e \rightarrow \infty, \ P \rightarrow 0 \)

\[
M = 0.85f'_{dc} \frac{ba(x - a/2)}{x} + f_{dy} \frac{d'}{x} A_s
\]

where

\[
a = 0.537 \quad d = 0.537(21) = 11.3 \text{ in.}
\]

\[
M = 0.85(39)(63)11.3(9.6 - 5.65)/12 + 52\left(\frac{17}{12}\right)6.24
\]

\[
= 777 + 460 = 1237 \text{ kip-ft}
\]

P vs M curve for rectangular section

Tension failure:

\[
A_s = 12.48 \text{ in.}^2
\]

\[
A_s' = 6.24 \text{ in.}^2
\]

\[
da = 20 \text{ in., } d' = 17 \text{ in.}
\]

\[
b = 24 \text{ in.}
\]

\[
P + A_s f_{dy} = A_s' f_{dy} + 0.85f'_{dc} \frac{ba}{x}
\]

\[
P + 12.48(52) = 6.24(52) + 0.85(3.9)(24) a
\]

\[
a = 0.01258 P + 4.08
\]

\[
M = 0.85f'_{dc} \frac{ba}{x} \left(\frac{b}{2} - \frac{d}{2}\right) + A_{s'dy} d' + (A_s - A_s') f_{dy} (d - t/2)
\]

\[
= 0.85(3.9) \frac{a}{12} (12 - a/2) + 6.24(52) \frac{17}{12} + 6.24(52) \frac{8}{12}
\]

\[
= 6.65 a (12 - a/2) + 460 + 216
\]

\[
= 6.65 a (12 - a/2) + 676
\]

Table 11.7. Tabulation of Ultimate Axial Load vs Ultimate Moment (Column-Rectangular Section at Top of Wall)

<table>
<thead>
<tr>
<th>( P ) (kips)</th>
<th>( a ) (in.)</th>
<th>( M ) (ft-kips)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4.08</td>
<td>946</td>
</tr>
<tr>
<td>50</td>
<td>4.71</td>
<td>978</td>
</tr>
<tr>
<td>100</td>
<td>5.34</td>
<td>1008</td>
</tr>
<tr>
<td>150</td>
<td>5.97</td>
<td>1034</td>
</tr>
<tr>
<td>200</td>
<td>6.60</td>
<td>1058</td>
</tr>
<tr>
<td>250</td>
<td>7.22</td>
<td>1079</td>
</tr>
</tbody>
</table>
Compression failure:

Axial load $A_c = b't - (A'_s + A_s) = 24^2 - (12.48 + 6.24)$

$= 576 - 19 = 557$

$P = A_c f'_{dc} + (A'_s + A_s) f'_d = 557(3.9) + 19.72(52)$

$e \to \infty, P \to 0$

$M = 0.85 f'_{dc}(0.537) d b \left( t/2 - 0.537d \right) + 676$

$= 0.85(3.9)(10.74)(24) \frac{1}{12} (6.63) + 676$

$= 473 + 676$

$= 1149$ kip-ft

Mass: Assume an effective earth mass of depth equal to one-half the column span

Mass, $m = \left[ \frac{1}{2} (10.5)(0.1)(10.5) \frac{63}{12} + \frac{1}{2} (10.5)(0.1)(10.5)^2 

+ \frac{14}{12} (0.150)(10.5) \frac{63}{12} + 2(2)(0.150)(10.5) \right] \frac{1}{32.2}$

$= \left[ 28.90 + 11.02 + 9.64 + 6.30 \right] \frac{1}{32.2}$

$= 1.728$ kip-sec²/ft

Net resistance $R = \left[ 0.60(2) + \frac{63}{12} (1.725) + 1.125(2) \right] \left( \frac{1}{2} \right) (10.5) \frac{1}{2}$

$= R - 33$

d. Determination of Maximum Deflection and Dynamic Reactions by Numerical Integration.

Elastic range:

$K_{LM} = 0.830$ (table 6.4 and figure 6.29)

$k_1 = \frac{294EI}{L^3} = 294(3000)26,600/(10.5)^3 \frac{144}{1} = 140,800$ kips/ft

$T_n = 2\pi \sqrt{\frac{m K_{LM}}{k_1}} = 2\pi \sqrt{\frac{1.728(0.83)}{140,800}} = 0.020$ sec

Use $\Delta t \approx 0.1 T_n$, $\Delta t = 0.0025$ sec

$\ddot{y}_{\Delta t}^2 = \frac{m K_{LM}}{K_{LM}} (P_n - R_n) = \frac{(0.0025)^2}{1.728(0.830)} (P_n - R_n)$

$= 4.36(10)^{-6} (P_n - R_n)$
\[ P_n = 0.144 \left( \frac{53}{12} \right)^{\frac{3}{2}} (10.5) \]  
\[ P_s = 3.97 \]  
\[ P_s \text{ kips} \]

\[ R_n = K_1 y = 140,800 \text{ y kips} \]

\[ R_{lm} = \frac{9.93}{10.5} M_{ps} = 0.945 M_{ps} \]

Elasto-plastic range:

\[ K_{LM} = 0.866 \text{ (table 6.4 and figure 6.27)} \]

\[ k_{ep} = \frac{6EI}{L^3} = 60(3000) \frac{31,080}{(10.5)^3} 144 = 33,600 \text{ kips/ft} \]

\[ \ddot{y} \Delta t^2 = \frac{(\Delta t)^2}{m K_{LM}} (P_n - R_n) = \frac{(0.0025)^2}{1.728(0.866)} (P_n - R_n) \]

\[ = 4.18(10)^{-6} (P_n - R_n) \]

\[ R_m = \frac{6}{10.5} (M_{ps} + M_{pc}) = 0.571 (M_{ps} + M_{pc}) \]

\[ M_{ps} \text{ and } M_{pc} \text{ are taken from the M-P curves as a function of the axial load on the wall column. The axial loads are obtained from the static and dynamic reactions of the girder and roof slab upon the wall and column.} \]

**Table 11.8. Determination of Maximum Deflection of Exterior Wall Column**

<table>
<thead>
<tr>
<th>t (sec)</th>
<th>Axial Load Top (kips)</th>
<th>Axial Load Center (kips)</th>
<th>R_{lm} (kips)</th>
<th>R_m (kips)</th>
<th>P_n (kips)</th>
<th>P_n - R_n (kips)</th>
<th>\ddot{y}_{n} t^2 (ft)</th>
<th>y_n (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>190</td>
<td>211</td>
<td>962</td>
<td>922</td>
<td>99.3</td>
<td>0</td>
<td>\frac{1}{2}(99.3)</td>
<td>0</td>
</tr>
<tr>
<td>0.0025</td>
<td>217</td>
<td>243</td>
<td>973</td>
<td>941</td>
<td>97.3</td>
<td>102.8</td>
<td>-5.5</td>
<td>0.00022</td>
</tr>
<tr>
<td>0.0050</td>
<td>217</td>
<td>243</td>
<td>973</td>
<td>941</td>
<td>97.3</td>
<td>102.8</td>
<td>-5.5</td>
<td>0.00033</td>
</tr>
<tr>
<td>0.0075</td>
<td>221</td>
<td>243</td>
<td>973</td>
<td>941</td>
<td>97.3</td>
<td>102.8</td>
<td>-5.5</td>
<td>0.00043</td>
</tr>
<tr>
<td>0.0100</td>
<td>217</td>
<td>243</td>
<td>973</td>
<td>941</td>
<td>97.3</td>
<td>102.8</td>
<td>-5.5</td>
<td>0.00043</td>
</tr>
<tr>
<td>0.0125</td>
<td>217</td>
<td>243</td>
<td>973</td>
<td>941</td>
<td>97.3</td>
<td>102.8</td>
<td>-5.5</td>
<td>0.00043</td>
</tr>
</tbody>
</table>

The maximum dynamic resistance developed, \( R = 194.1 \text{ kips} \), is less than the corresponding \( R_{lm} \), hence the column section is more than adequate in bending.

Dynamic reaction for bond and shear:

\[ V = 0.44 R + 0.06 P \]

\( (V_{\text{maximum}})_{\text{dynamic}} \) occurs at 0.010 sec
\[
(V_{\text{maximum}})^{\text{dynamic}} = 0.44 \,(194.1) + 0.059 \,(95.3) = 91.0 \,\text{kips}
\]

\[
V_{\text{maximum}} = V_{\text{dynamic}} + V_{\text{static}} = 91.0 + R_{\text{top}}
\]

\[
R_{\text{top}} = \frac{63}{12} \left[ \frac{1}{10.5} \,(0.6) \frac{10.5^2}{2} + (0.525) \frac{10.5^2}{6} - 8.41 + 7.45 \right] = 20.9 \,\text{kips}
\]

\[
V_{\text{maximum}} = 91.0 + 20.9 = 111.9 \,\text{kips}
\]

e. **Shear Strength and Bond Stress.**

\[
v' = \frac{V}{bjd} = \frac{111,900}{24(0.88)20} = 265 \,\text{psi}
\]

\[
v_{\text{allowable}} = 0.04 \,f_c' + 5000 \,p \quad (\text{eq 4.24b})
\]

\[
= 0.04(3000) + 5000 \,(0.0248) = 244 \,\text{psi}
\]

Use #3 ties at 12 in.

Bond, \(u_{\text{allowable}} = 450 \,\text{psi} \]

\[
\Sigma_{\text{req'd}} = \frac{V}{ujd} = \frac{111,900}{450} (0.88) 20
\]

\[
= 14.11 \,\text{in.} < 4.43 \,(8) = 35.4 \,\text{in. furnished}
\]

Therefore, bond is OK.

**11-22 DESIGN OF EXTERIOR WALL COLUMN FOOTING.**

a. **Static Design.**

Assume a footing 5 by 5 by 2.5 ft.
Column load:

Overburden and surcharge = 20(10)11(0.100) = 220.0 kips
Roof weight = \( \frac{14}{12} \) 0.150(20)10 = 34.9
Girder weight = \( \frac{30}{12} \) 2(0.150)10 = 7.5
Column weight = 9(2)2(0.150) = 5.4
Footing weight = 5(5)2.5(0.150) = 9.4
Overburden on extension of footing
= 5(1.5)22.17(0.1) = 16.6
Surcharge on extension of footing = 5(1.5)0.1 = 0.8
Concrete floor weight = \( \frac{6}{12} \) 0.150(20)9 = 13.5
Weight of gravel = \( \frac{6}{12} \) 5(1.5)0.1 = 0.4

308.5 kips

Footing moment (clockwise positive):

Moment due to lateral pressure = \( (1.125)(8)(2)(0.96) = +16.8 \) kip-ft
Moment due to overburden on footing = -16.6(1.75) = -29.0
Moment due to surcharge on footing = -0.8(1.75) = -1.4
Moment due to gravel on footing = 0.4(1.75) = +0.7
Moment due to weight of floor = 13.5(1.00) = +13.5

Moment = +0.6 kip-ft

For static design, assume moment = 0

Bearing pressure = \( \frac{P}{A} = 308.5/25.0 = 12.3 \) kips/ft\(^2\) OK.

Footing shear not critical since footing is small

Footing moment at column face, interior face

\[ M = \frac{w_{\text{pressure}} L^2}{2} - \frac{w_{\text{D.L.}} L^2}{2} = 12.3(1.5)\frac{1.5}{2} - (0.5)1.5\left(\frac{1.5}{2}\right) \]

= 13.2 kip-ft

Take d = 26 in.

\[ A_b = 0.85 \frac{M}{f_{sb} d} = 0.85 \frac{13.2}{20(0.88)26} = 0.29 \text{ in.}^2/\text{ft} \]

Use \#5 bars 12-in. o.c. each way.

b. Dynamic Investigation.
Axial load:

Dynamic girder reaction = 481.0 kips
Overpressure over girder area = \( \frac{1}{2} \times 19(22.2)0.144 \) = 30.4
Overpressure on footing extension = 5(1.5)22.2(0.144) = 24.0

Static load = 308.5 kips

\[ \frac{643.9}{8} \] kips

Moment (positive clockwise):

Static moment = +0.6 kip-ft

Moment due to lateral overpressure = \( \frac{1}{12}wL^2 \)
= \( \frac{1}{12} \times 22.2(0.144)10.5^2 \) = +29.4

Moment due to lateral overpressure on footing = 24.0(1.75)

Total moment = -12.0 kip-ft

Bearing pressure

\[ P_{\text{max}} = \frac{P}{A} + \frac{Mc}{I} = \frac{643.9}{25} + \frac{12.0(6)}{(5)^3} \]

= 33.8 + 0.6 = 34.4 kips/ft^2 > 30, therefore NG.

Try 6- by 6- by 2.5-ft footing

Axial load:

Dynamic girder reaction = 481.0 kips
Overpressure over girder area = 30.4
Overpressure on footing extension = 6(2.22)(0.144) = 38.4

Overburden; surcharge; roof, girder, and column weight = 267.8

Footing weight = 6(6)2.5(0.150) = 13.5

Overburden on footing extension = 6(2)22.17(0.1) = 26.6

Concrete floor weight = 13.5

Weight of gravel = \( \frac{6}{12}(6)2(0.1) \) = 0.6

\[ \frac{871.8}{8} \] kips
Moment:

Moment due to lateral overpressure = +29.4 kip-ft

Moment due to lateral overpressure on footing = -38.4(2) = -76.8

Momnet due to lateral earth pressure = +16.8

Moment due to overburden on footing = 26.6(2) = -53.2

Moment due to gravel on footing = 0.6(2) = +1.2

Moment due to weight of floor = 13.5(1.0) = +13.5 -69.1 kip-ft

Bearing pressure:

\[ P_{\text{max}} = \frac{871.8}{36} + \frac{69.1(6)}{(6)^3} \]

\[ = 24.2 + 1.9 \]

\[ = 26.1 \text{ kips/ft}^2 \text{ at outside edge} \]

\[ = 22.3 \text{ kips/ft}^2 \text{ at inside edge} \]

Moment at inside face equals approximately

\[ M = 23(2)1 = 46 \text{ kip-ft} \]

Moment capacity:

Using \#5 bars at 12-in. o.c.

\[ M_p = 0.33(52)\left(\frac{26}{12}\right)\left[1 - \frac{0.33(52)}{12(26)1.7(3.9)}\right] \]

\[ = 34.7 \text{ kip-ft} \]

Therefore use \#4 bars 5-1/2-in. o.c.

\[ M_p = 49 \text{ kip-ft}; 46 \text{ kips} < 49 \text{ kips}, \text{ therefore OK.} \]

Bearing capacity of wall:

Area = \( \frac{14}{12}(1) = 1.17 \text{ ft}^2/\text{ft} \)

Weight of wall = \( \frac{14}{12}(11)0.150 = 1.9 \text{ kips/ft} \)

Dynamic roof reaction = 20.8 kips/ft

Total axial load = 22.7 kips

Bearing stress = \( \frac{P}{A} = \frac{22.7(12)}{14} = 19.5 \text{ kips/ft}^2 \), OK.

11-23 DESIGN SUMMARY. Figure 11.12 shows the location of the various elements that were designed in detail in this example. Sections are taken...
Figure 11.12. Perspective showing locations of designed sections

throughout the building to indicate the final design, and reference is given to the paragraphs where each design is first presented. Since the sketches show only designed reinforcement additional nominal reinforcement (temperature, etc.) is to be added by the detailer.
BIBLIOGRAPHY

1. American Concrete Institute, Building Code Requirements for Reinforced Concrete. (ACI 318-56) Detroit, Michigan, July 1956.

2. American Concrete Institute, Committee 317. Reinforced Concrete Design Handbook. Detroit, Michigan.


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